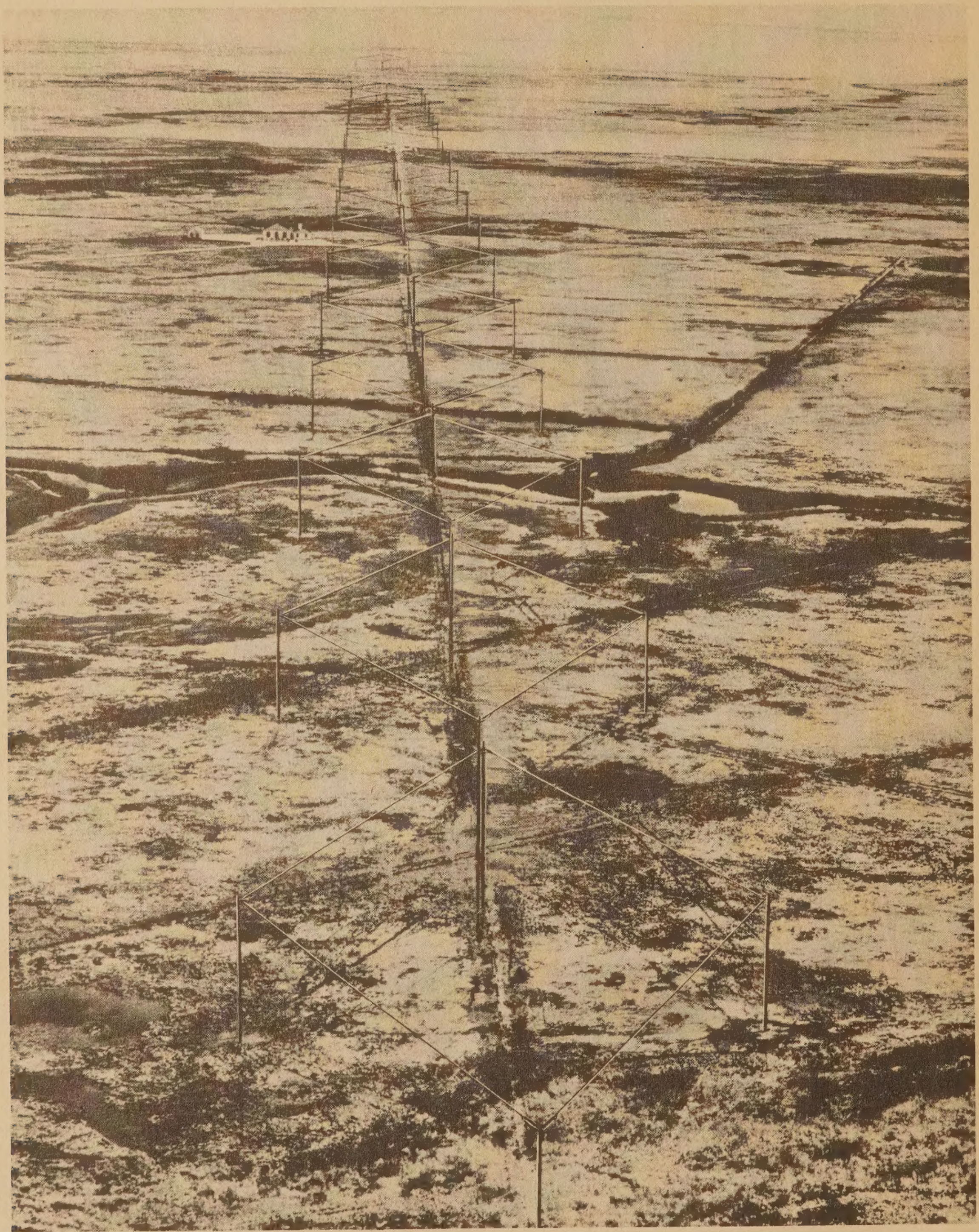
Howard S. Smith

RHOMBIC ANTENNA DESIGN



Array of rhombic antennas, American Telephone and Telegraph transatlantic radiotelephone receiving station.

RHOMBIC ANTENNA DESIGN

BY

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FOREWORD

When there was built in 1929 at Lawrenceville, New Jersey, a radio telephone station for initiating overseas short-wave service, the most pictured feature of the new establishment was a gigantic wire fence or net, a mile long, stretched across the landscape on a row of 185 foot towers. This comprised the transmitting antenna complement for the three telephone circuits to Europe.

A year ago the nets were taken down, the towers dismantled and sold for junk. Near them had arisen a number of telephone poles carrying at odd looking angles a few almost invisible wires. While this was the most spectacular conquest of the rhombic antenna, it is not the only measure of its impact on short wave radio telephony. Subsequent stations constructed in Florida and California made use of this simple, efficient radiator from the start and its economy was a factor in justifying the establishment of the new routes to South America, Hawaii and the Orient.

Although the rhombic has been known since its first published description by E. Bruce in 1931, it has not been widely adopted outside the radio telephone field of the Western Hemisphere. One reason for this, no doubt, is the lack of easily usable information on the rational design of this antenna to fit it in each case to the work to be done - and a sound design is necessary to successful use of any directive system.

To make design information available to radio telephone engineers of the Bell System, A. E. Harper compiled this handbook from the published and unpublished work of his colleagues in Bell Telephone Laboratories. The prospect that libraries, students and engineers in other departments of the radio art would like to have this material has prompted its formal publication.

RALPH BOWN

Bell Telephone Laboratories

PREFACE

This text has been compiled for the convenience of engineers engaged in the design of short wave radio communication circuits. For this reason emphasis has been placed upon current design and construction practices, rather than upon basic theory.

In order to meet the requirements of the practical designer, it includes an introductory discussion of directional radio transmission, followed by a description of horizontal rhombic antenna design methods and mechanical construction practices. Tabulated and graphical functions found useful in antenna development work have been included to expedite computation. The plans of typical transmitting and receiving antennas have been appended to indicate the overhead wire line hardware ordinarily required.

The subject matter has been collected from the published and unpublished reports of many engineers associated with the development and construction of directional antennas in the Bell Telephone System. Since so large a number of investigators is concerned, it is impractical to make personal acknowledgment for each contribution. The publications cited in the bibliography have been selected solely from the standpoint of furnishing modern information on antenna design. They are in no way indicative of priority, nor are they intended to give a historical account of the development of the art. While much material has been taken from the papers cited, the greater part of the design data given has been obtained from unpublished memoranda.

A. E. HARPER

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ILLUSTRATIONS

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1. General Discussion of Directional Antennas

It was realized very early in the history of radio communication that substantial improvements in transmission could be obtained by limiting radiation or reception to the small solid angle which includes the direction of the transmission path[1]. By this means it is possible to produce a required field strength at the receiving station without prodigally radiating power in undesired directions. Proportionately important advantages may be realized by the use of directional receiving antennas, for noise arriving from directions not included in the solid sector of desired receptivity is reduced in proportion to the directional discrimination of the system. The rapid advances in transoceanic radio communication have been closely paralleled by the development of the various forms of directional antenna systems, including coil antennas[1], arrays of verticals[2,3,4], wave antennas[5], rhombic antennas[6,7], and arrays of rhombics[8]. The economy of transmitter power resulting from the use of directional transmitting and receiving antennas has in many cases permitted the operation of radio circuits which would otherwise have been impractical[5,15,29].

2. Antenna Gain

Fundamentally the advantage to be derived from a directive transmitting antenna is the power economy effected by confining the radiation to a limited solid angle. Such a directive or beam characteristic is obtained by so shaping and locating the units of the antenna system that the vector addition of waves from various parts of the system will produce reinforcement in a desired sector, and cancellation in undesired directions[4]. The completeness of this reinforcement and cancellation is often a function of the physical size of the antenna structure, and the antenna may represent an investment comparable with that of the transmitting apparatus.

The percentage of the time that a radio circuit is available for commercial service depends principally upon the maintenance of a satisfactory signal-to-noise ratio.[17] This in turn is governed by the transmitter power and the effectiveness of the transmitting and receiving antennas[15]. It is considered good practice to minimize the total annual charges by proportioning the investment in apparatus, and in transmitting and receiving antennas so that the annual expense of a given incremental signal-to-noise improvement will be approximately the same, whether obtained by increases in transmitter power, or in the effectiveness of either antenna system[6].

A convenient numeric for stating the effectiveness of a directive transmitting antenna is obtained from the ratio of the power consumed by a reference antenna of known characteristics to the power consumed by the antenna in question, when equal field intensities are produced at a specified distant point[34]. This ratio expressed in decibels has been called the signal gain of the antenna[4]. Likewise for receiving antennas, the signal gain is the ratio of the power at the directional antenna terminals, to that at the terminals of a reference antenna when the same signal field is applied to each.

It is obvious that optimum transmission can only be obtained when the horizontal and vertical angles of maximum antenna radiation and reception coincide with the corresponding horizontal and vertical angles of optimum radio transmission. Since the vertical angles of the optimum radio transmission path are a function of path length, frequency[28] and ionized layer height[13,14], and deviations of path azimuth often occur, measurements of antenna signal gain must be made over the radio path for which the circuit was designed, and should cover an extended period substantially undisturbed by magnetic storms[8,22].

FIGURE I

ANTENNA GAINS AND PATTERNS

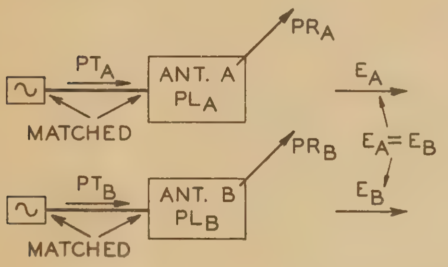
GS = SIGNAL GAIN, GD = DIRECTIVITY GAIN, E = FIELD INTENSITY

(Hypothetical Antenna with Spherical Directional Pattern and No Heat Loss has Reference Gain)

TRANSMITTING CASE:

PT = Input Power, PR = Radiated Power, PL = Heat Loss, $L = 10 \text{ Log } \frac{PT}{PR}$ db

PT = PR + PL



By Definition:

$$\frac{GD_A}{GD_B} = \frac{PR_B}{PR_A} \qquad (1)$$

$$\frac{GS_A}{GS_B} = \frac{PT_B}{PT_A} \qquad (2)$$

From (1) and (2) we obtain:

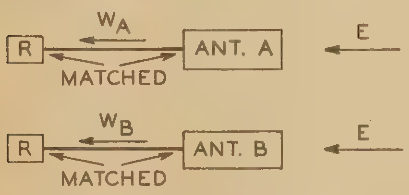
$$\frac{GS_A}{GS_B} = \frac{GD_A}{GD_B} \times \frac{PT_B/PR_B}{PT_A/PR_A} \qquad (3)$$

or in db Figures:

$$GS_A - GS_B = GD_A - L_A - [GD_B - L_B] \qquad (4)$$

RECEIVING CASE:

W = Received Power.



By Definition:

$$\frac{GS_A}{GS_B} = \frac{W_A}{W_B} \qquad (5)$$

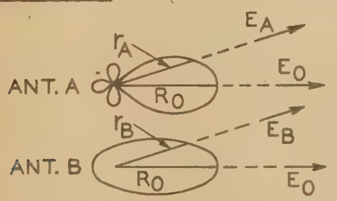
The Reciprocity Theorem Gives:

$$\frac{W_A}{W_B} = \frac{PT_B}{PT_A} \qquad (6)$$

(2),(5) and (6) show that the transmitting and receiving signal gains of an antenna are alike. (An equation corresponding to (4) does not exist).

DIRECTIONAL PATTERNS

TRANSMITTING CASE:



E = Field Intensity in Different Directions at a Great Distance.

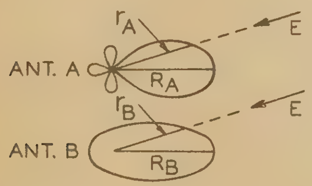
By Definition:

$$\left.\begin{array}{l} r_A = K_T \times E_A \\ r_B = K_T \times E_B \end{array}\right\} \text{Where KT is a constant.} \qquad (7)$$

From (1) we obtain:

$$\frac{GD_A}{GD_B} = \frac{\int r_B^2 d\varphi}{\int r_A^2 d\varphi} \qquad (\varphi \text{ is the solid angle}) \qquad (8)$$

RECEIVING CASE:



E = Const. = Field Intensity of Waves Arriving From Different Directions.

By Definition:

$$\left.\begin{array}{l} r_A = KR \sqrt{W_A} \\ r_B = KR \sqrt{W_B} \end{array}\right\} \text{Where KR is a constant and W the received power.} \qquad (9)$$

From the Reciprocity Theorem it Follows that the shapes of transmitting and receiving patterns of an antenna are similar. Using receiving patterns as defined, equation (8) becomes:

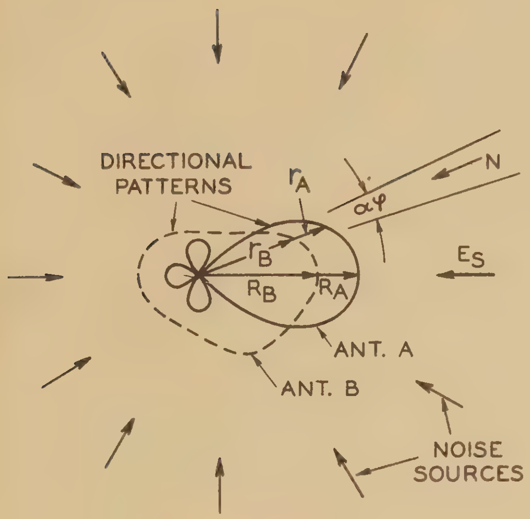
$$\frac{GD_A}{GD_B} = \frac{R_A^2}{R_B^2} \times \frac{\int r_B^2 d\varphi}{\int r_A^2 d\varphi} \qquad (10)$$

The attached Figure 1 shows that the transmitting signal gain of an antenna is equal to its receiving signal gain. It does not necessarily follow, however, that equivalent improvements in signal-to-noise ratio may be obtained by a given increase in the signal gain at either the transmitting or receiving antennas. Little discussion is required to show that an increase in the transmitting antenna signal gain proportionately increases the amplitude of the received signal relative to received noise and receiving apparatus noise, and thereby increases the signal-to-noise ratio. At the receiving station, however, if we could increase the signal gain and hold the directional discrimination constant, we would increase the amplitude of the signal relative to the receiving apparatus noise but not improve the ratio of signal to received radio noise[16]. On the other hand, if we decrease the receptivity of the antenna in undesired directions relative to reception in the desired direction, radio noise arriving from these angles will be reduced relative to the desired signal[6]. A somewhat more explicit explanation of the part played by directivity gain in the reduction of received noise is shown on the attached Figure 2.

Quantitatively the directivity gain of a directional antenna, referred to a completely non-directional antenna, would be the ratio of the power per unit solid angle flowing in the direction of maximum radiation or reception, to the average power per unit solid angle flowing in all directions to or from the antenna. In this discussion we shall define the unit solid angle as the angle subtended by a unit area on a sphere of unit radius circumscribed about the sending or receiving antenna in question. Unfortunately the difficulty of integrating the directional characteristic prevents an analytical computation of directivity gain in many practical cases.

FIGURE 2

SIGNAL TO NOISE RATIO & DIRECTIVITY GAIN



The energy received from a single noise source N (for instance a single lightning discharge) is proportional to r^2. The energy received from several noise sources is the sum of the energies received from each source.

UNIFORM DISTRIBUTION ASSUMPTION:

The energy received in a period T* from the noise sources located in a unit solid angle is constant in all directions. Then:

$$dQN_A = KN \times r_A^2 \times d\varphi$$

$$dQN_B = KN \times r_B^2 \times d\varphi$$

Where dQN is the energy received in a period T from the noise sources located in the solid angle $d\varphi$ and KN a constant.

By summation:

$$\left.\begin{aligned} QN_A &= KN \int r_A^2 d\varphi \\ QN_B &= KN \int r_B^2 d\varphi \end{aligned}\right) \qquad (2)$$

The energies received in a period T from the signal E_S are:

$$\left.\begin{aligned} QS_A &= KS \times T \times R_A^2 \\ QS_B &= KS \times T \times R_B^2 \end{aligned}\right) \text{Where KS is a const.} \qquad (1)$$

From (1) and (2) we obtain the average signal to noise ratio improvement of antenna A over antenna B:

$$\frac{S/N_A}{S/N_B} = \frac{R_A^2}{R_B^2} \times \frac{\int r_B^2 d\varphi}{\int r_A^2 d\varphi} \qquad (3)$$

From (3) and [(10) Fig. 1] it follows that

$$\frac{S/N_A}{S/N_B} = \frac{GD_A}{GD_B} \qquad (4)$$

or the S/N improvement of a receiving antenna is equal to its directivity gain.

The signal gains and directivity gains are equal for antennas without heat loss. (4) shows that <u>all</u> antennas without heat loss have identical noise outputs.

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*T may be several hours or days when local thunderstorms upset the uniform distribution of noise sources.

If we assume a random or omnidirectional distribution of radio noise sources about a receiving station, it is apparent from the above definition that the average signal-to-received-noise ratio will be proportional to the directivity gain of the antenna. Random distribution of received noise is not often found in practice on account of the existence of localities of high thunderstorm frequency, whose positions and effects vary with season and hour. Measurements continued over long periods show that a close agreement between directivity gain and the average signal-to-noise ratio actually exists[9].

Briefly summarizing the above discussion, the fundamental purpose underlying the use of ordinary directive antennas is the attainment of high signal-to-noise ratios. High signal gain at the transmitting antenna reduces the effect of received noise and receiving apparatus circuit noise. High signal gain at the receiving antenna reduces the effect of apparatus noise, and high directivity gain reduces the effect of external radio noise sources.

3. Antenna Location

When choosing a site for any form of directive antenna it is desirable to avoid obstructions such as mountains or conducting structures which lie directly in the radio transmission path. Level or uniformly sloping ground is ordinarily assumed in the design formulas and this type of topography gives results which can be accurately predicted[9,12,44] Sources of natural noise such as annual thunderstorm concentrations and mountain ranges of high thunderstorm occurrence should not be in the radio transmission path near the receiving station if it is possible to avoid them.

Receiving antennas must be located in places substantially free from sources of man-made radio noise[(44)]. It is obvious that such sources of noise when located in the radio transmission path of the received signal, are of considerably more importance than noise sources to the rear, or in comparatively insensitive sectors of the directional characteristic.

Rural sites have been found satisfactory for the avoidance of the disturbances usually associated with large cities, such as miscellaneous industrial sources, electrical therapy and diathermy apparatus, electrical railway stopping and starting transients, and many other continuous and intermittent noises. Even in the more thinly populated districts consideration must be given to the disturbances arising on motor highways, power boat channels and power transmission lines[16].

The resultant radiated or received signal field is the vector sum of a direct ray and a ray reflected from the ground. Since a level and substantially uniform ground plane is assumed in the theory underlying a practical design, performance in agreement with theory can only be expected when actual ground conditions approximate those assumed[12]. Deviations from the predicted vertical plane directivity of existing antennas have been traced to topography or unexpected ground properties.

4. Horizontal Orientation

An antenna is ordinarily oriented so that its azimuth of maximum radiation or reception lies in the plane of the great circle transmission path joining the transmitting and receiving stations. Recent studies of transatlantic radio transmission[22] show that rather wide departures from a great circle transmission path occur at times of disturbed transmission, and when the path is partially or wholly blanketed by darkness. To avoid discrimination against desired signals which depart from a great circle path, and to preserve a reasonable suppression of undesired signals, a directive characteristic with a broad transmission sector and a suppression of secondary lobes in excess of 8-12 db has been found more to be desired than a characteristic with an extremely sharp major lobe.

Preliminary estimates of the azimuth of the great circle transmission path, and data on the approximate geographical coverage of the horizontal transmission sector of the

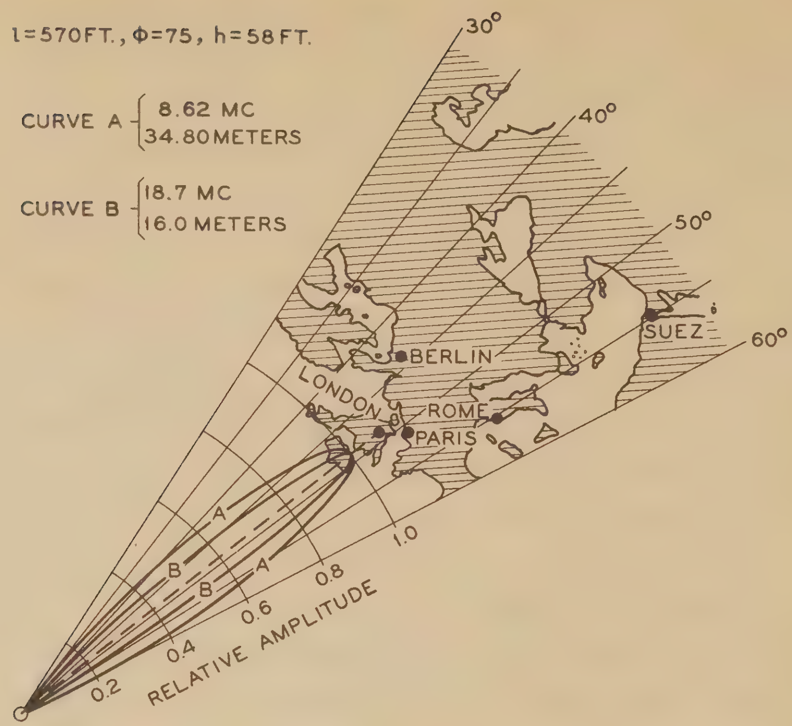
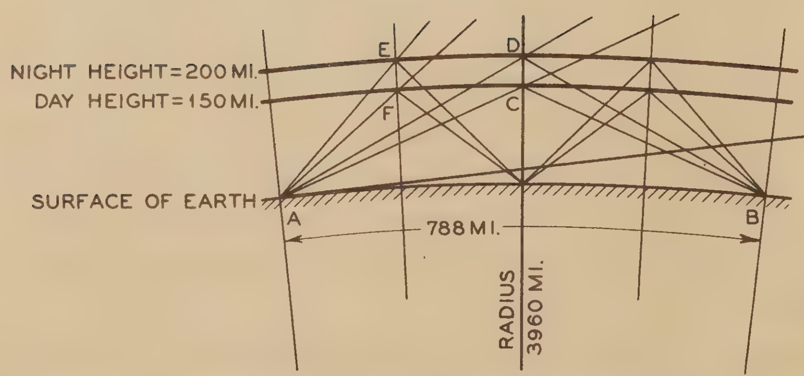


FIG. 3 — BEARING OF PRINCIPAL EUROPEAN CITIES IN APPROXIMATE RELATION TO THE DIRECTIVITY OF A TRANSATLANTIC TYPE RHOMBIC ANTENNA



ANGLE	DEG.
TAC	17.5
TAD	22.9
TAF	34.4
TAE	42.5

FIG. 4 — TRANSMISSION PATHS LAWRENCEVILLE - BERMUDA

directional characteristic may be obtained either from azimuthal projection maps or from great circle sailing charts. Unfortunately azimuthal maps are only accurate for bearings taken from locations near the point for which the map was designed and are available for azimuths about Washington, D.C. and San Francisco, California[31]. The attached Figure 3 is based on this type of map projection. The great circle sailing charts or gnomonic projections published by the U. S. Hydrographic Office, have a more general utility because they may be used to find the great circle bearing to or from any point shown on the map. They have the slight disadvantage that although all great circles appear as straight lines on the chart, reference must be made to a simple geometrical construction to determine the azimuth or distance. These charts are available for the North and South Atlantic, the North and South Pacific and the Indian Oceans[35].

The great circle distance and the exact azimuth of the radio transmission path extending between two radio stations "A" and "B" at the opposite ends of the path, may be determined by spherical trigonometry without special map projections, using equations (1), (2) and (3).

Let L_A = Latitude of station A, positive for N lat., negative for S lat.

L_B = Latitude of station B, positive for N lat., negative for S lat.

$L_{o(AB)}$ = Longitude difference between A and B.

C_A = Direction of B from A, degrees E or W from north in Northern Hemisphere and from S in Southern Hemisphere.

C_B = Direction of A from B.

D_{A-B} = Great circle distance in minutes of arc or nautical miles (1 minute of arc = 1 nautical mile = 1.853 KM = 1.152 statute miles)

(1) $\cos D_{A-B} = \sin L_A \cdot \sin L_B + \cos L_A \cdot \cos L_B \cdot \cos L_{o(AB)}$

(2) $\sin C_A = \cos L_B \cdot \mathrm{CSC}\, D_{A-B} \cdot \sin L_{o(AB)}$

(3) $\sin C_B = \cos L_A \cdot \mathrm{CSC}\, D_{A-B} \cdot \sin L_{o(AB)}$

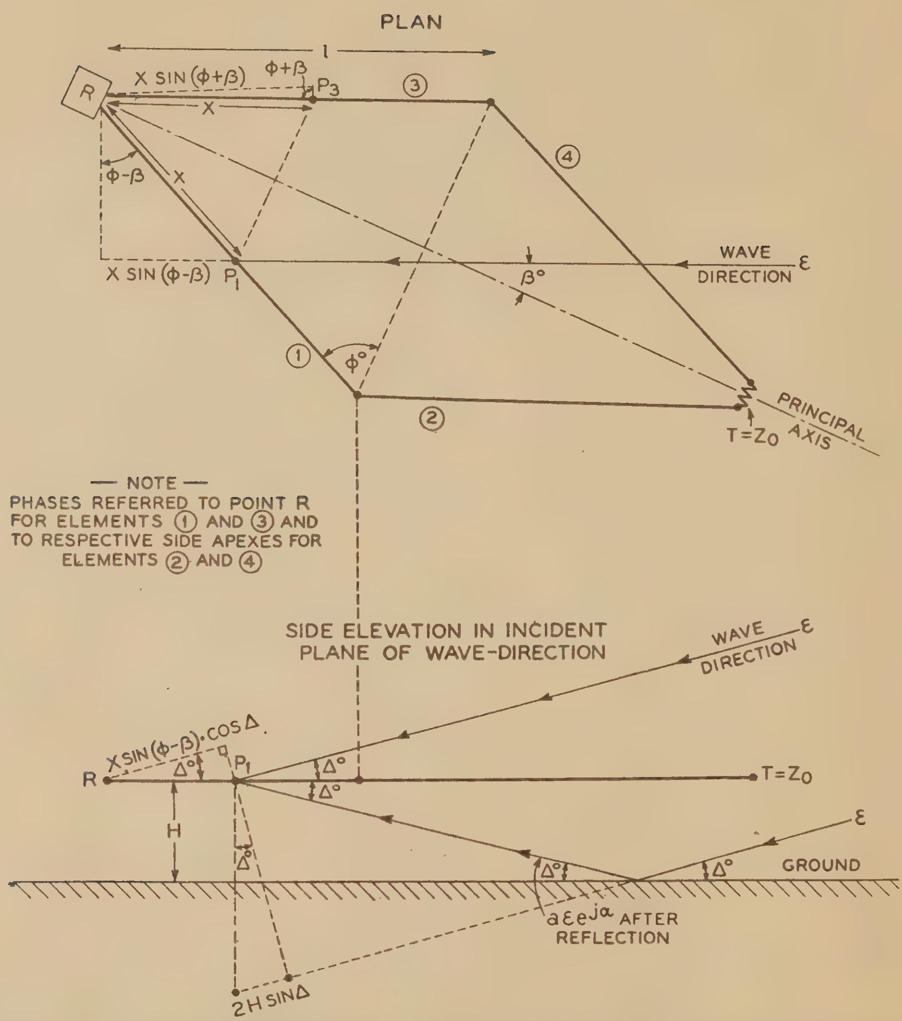


FIG. 5 - HORIZONTAL RHOMBIC ANTENNA DIMENSIONS

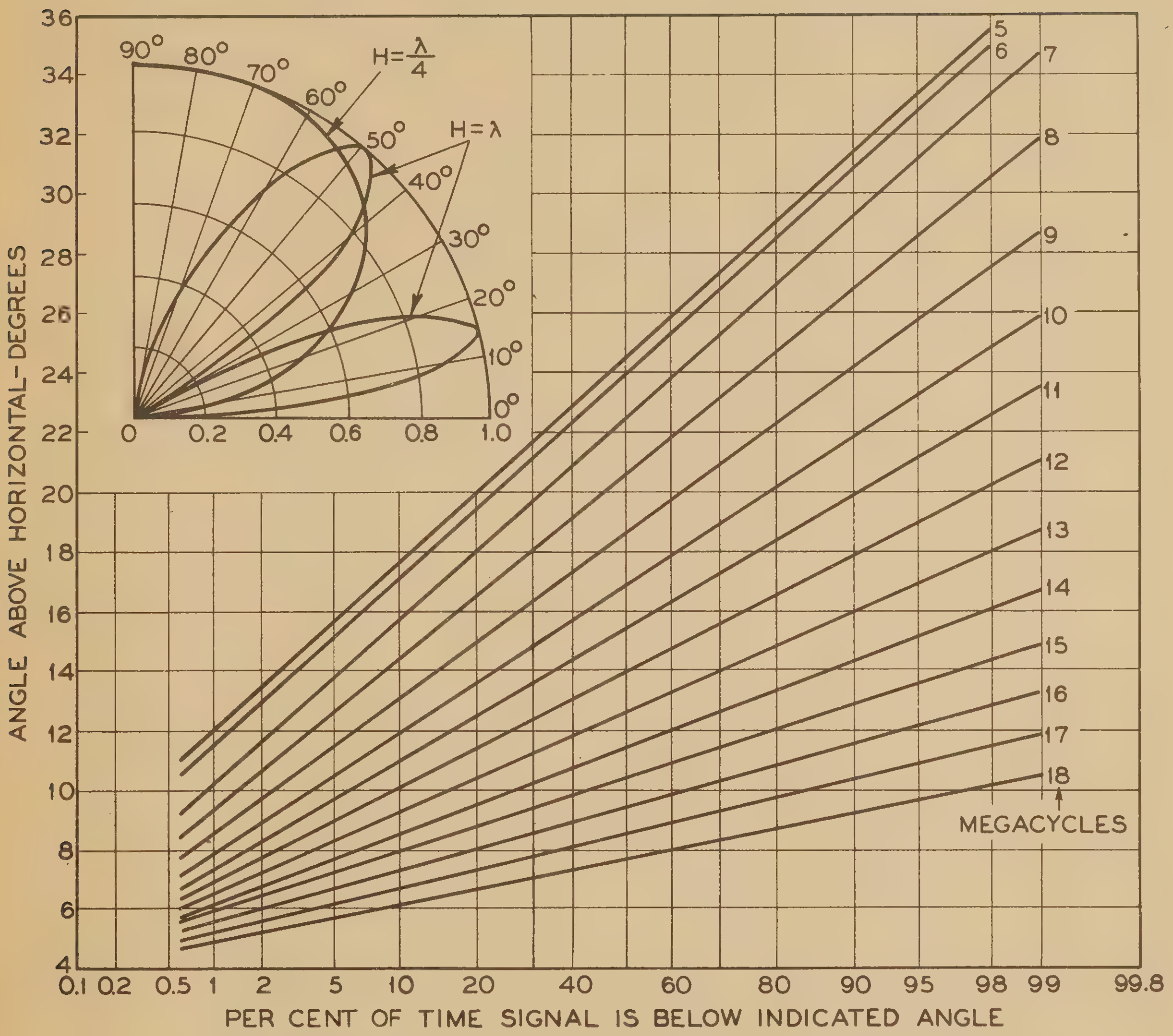


FIG. 6 – DISTRIBUTION OF MEAN VERTICAL ANGLES OF ARRIVAL
RUGBY – HOLMDEL TRANSMISSION PATH, 0300 – 1300 GMT
ONE DAY PER WEEK MAY 1933 – MAY 1934

5. Vertical Angles of Arrival and Departure

The vertical angle of maximum antenna sensitivity is ordinarily prealigned to coincide with the mean direction of arrival of a desired wave group. For transmitting antennas it is common practice at present to make the "angle of fire" equal to the angle of arrival used for the design of a corresponding receiving antenna for communication over the same transmission path.[42]

Methods for the measurement of the angles of arrival are beyond the scope of this discussion, but are covered in detail by Friis, Feldman and Sharpless[13]. An example of the statistical distribution of the mean vertical angles of arrival observed on the transatlantic path between Rugby, England and Holmdel, New Jersey for one day per week between 0300 and 1300 G.M.T. from May 1933 to May 1934, is plotted on Figure 6. It is known that the angle of arrival changes with the hour and season and it is probable that a long period variation exists[28].

Experience with 4 to 20 megacycle radio circuits has shown that in general the optimum vertical path angles are low when the path lengths exceed 2500 miles. Antennas intended for communication over great distances are therefore often designed to give maximum transmission or reception at 0 - 10 degrees above the horizon plane at the highest operating frequency. At lower frequencies the vertical path angles are generally greater as shown statistically on Figure 6. Due to the inherent properties of horizontal rhombic antennas the vertical angle of maximum transmission is an inverse function of the frequency, as shown progressively on Figures 24 - 29, inclusive. The antenna vertical sector of maximum transmission therefore follows the transmission path angle as the frequency is reduced.

At distances shorter than 2500 miles it is desirable to give special attention to the determination of the optimum vertical path angles. Lacking exact measured data, the path

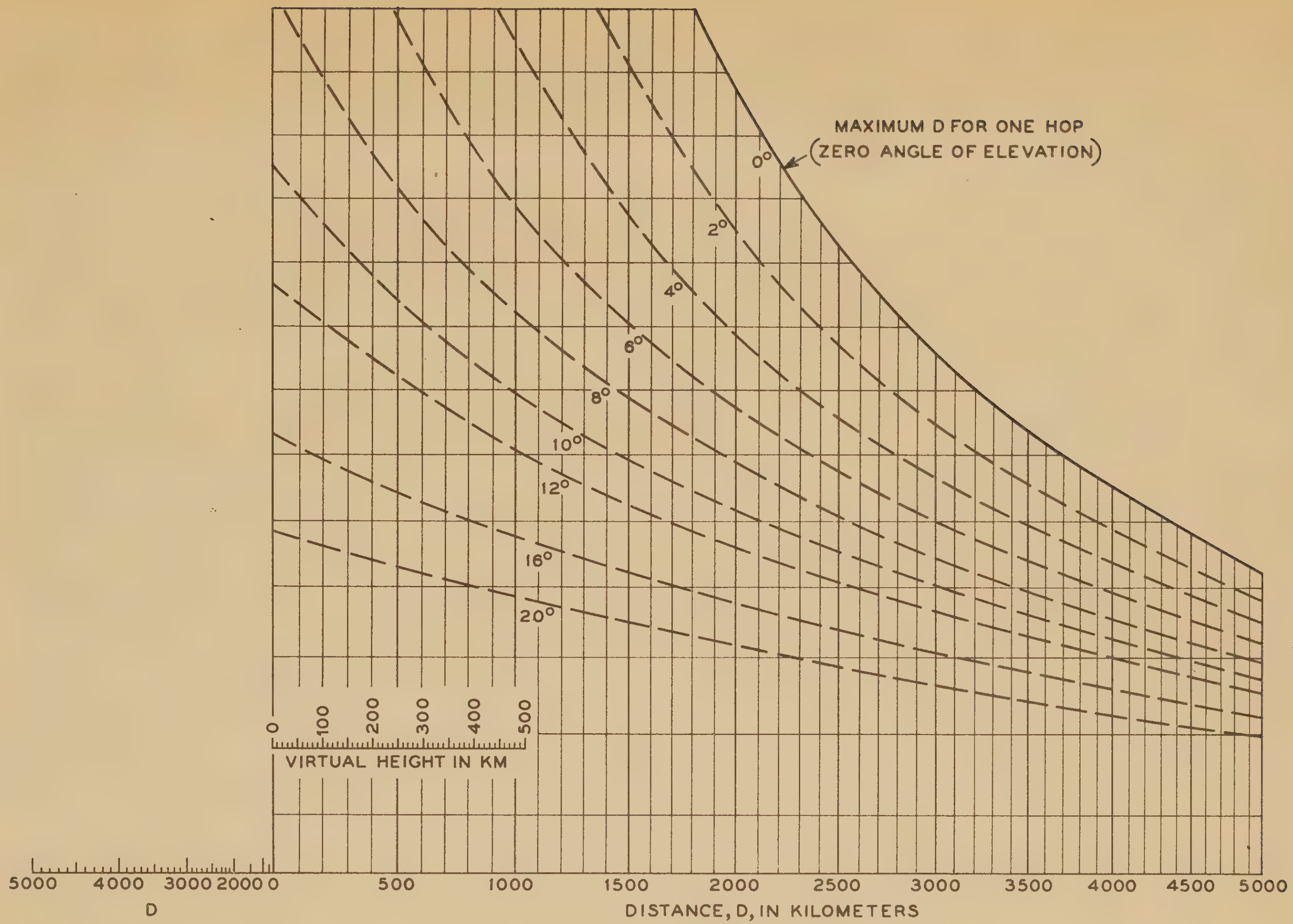


FIG. 7 — ALIGNMENT CHART FOR DETERMINING Δ, THE ANGLE WHICH THE RAY MAKES WITH THE HORIZONTAL AT THE TRANSMITTER OR RECEIVER. THE CURVED DASHED LINES ARE LINES OF EQUAL Δ

angles may be estimated by dividing the transmission path into a number of skips between the earth and the reflecting ionized layers[8,41]. A sample of this type of estimate is shown on Figure 4.

A graphical method for estimating the angle of arrival[28] from published monthly information[30] on the "Characteristics of the Ionosphere at Washington, D.C." has been recently devised by Newbern Smith of the U.S. National Bureau of Standards, and is shown on Figure 7. To use this chart pass a line or straight edge through the virtual height of the ionized reflecting layer, and the distance as measured with increasing distance to the left. The intersection of this line with the ordinate corresponding to the distance measured at the bottom of the chart with increasing distance to the right, determines a point which falls on or between the curves of equal vertical path angle. If this operation is repeated for one half the distance, one third the distance, etc., we will have the angle of arrival or departure for single, double and triple hop waves. The relative importance of these higher multiple skip waves depends upon the transmission frequency and the properties of the ionized layer at the time under consideration. A high angle requires a greater number of hops and reflections, each of which has a transmission loss which increases with the frequency.

Since the optimum transmission[41] frequency and associated vertical path angles vary with the hour and season, continuity of transmission requires the use of several operating frequencies. The ideal antenna therefore must meet the circuit gain requirements at the operating frequencies and have transmission sectors corresponding to the vertical path angles. To approach this ideal it is customary to determine the time distribution of the vertical path angles at different frequencies, and to design the antenna for a vertical angle which corresponds to the path angle during the heaviest

traffic period, making use of all operating frequencies as effectively as possible.

On account of the hourly and seasonal variations in the virtual height of the ionized layers, and the tendency of the transmission path to suddenly change from one number of skips to another, it is often considered advantageous to make the principal lobe of the vertical directivity diagram relatively broad. This expedient avoids discriminating against desired transmission at varying vertical angles but sacrifices gain at the lower frequencies. In practice it has been found that for transoceanic transmission rhombic antennas approximately one wavelength above the ground with sides 6 wavelengths long at the highest operating frequency give a satisfactory compromise. An alternative design is an antenna of variable directivity[14] or a musa array[8].

6. Properties of Horizontal Rhombic Antennas

The horizontal rhombic antenna provides an economical means for obtaining signal and directivity gains comparable with those secured by other more costly antennas of limited frequency range. The first and annual costs of single rhombic antennas compare favorably with those of known types of single channel arrays, and the rhombic type has the advantage that although optimum performance is obtained only in the neighborhood of a predetermined frequency, it will operate reasonably satisfactorily over a wide range of adjacent frequencies. The low cost, in addition to the ease of design, construction, and maintenance of the rhombic type has led to its almost universal use for point-to-point radio communication in the Bell System.

The wide frequency range obtained with a horizontal rhombic antenna when used for either transmitting or receiving, is due primarily to the fact that it approximates an aperiodic system with a relatively constant input impedance. This is supplemented by the fortunate coincidence that the trend of

vertical angle variation with frequency of the antenna, is the same as that of the mean vertical angle of arrival or departure of a long radio transmission path.

It will be shown later that a receiving horizontal rhombic antenna is unaffected by vertically polarized waves originating in the plane of the antenna or in a vertical plane through its axis. This property is of great service in reducing the interference caused by ignition systems. The horizontally polarized component of the noise is rapidly attenuated in traveling parallel to the ground, and the vertically polarized component arrives very nearly in the plane of the antenna and therefore produces little or no interference.[43]

7. Effect of the Ground Plane

Before considering in detail the directivity equations of a horizontal rhombic, a little attention will be devoted to the effect of the ground plane over which the antenna is erected. If an antenna could be constructed in free space, it would only be necessary to examine the directional behavior of a single directly transmitted ray between the radio transmitter and receiver. Actually however, the radiated field observed at a distant point, or the resultant field, received from a distant transmitter, is the vector sum of a directly transmitted field and a component reflected from the ground in front of the antenna. The reflected component is modified in amplitude by the ground losses, and in phase by both the phase change at reflection and the phase angle due to the difference in the length of the transmission paths of the two components. Adding the two vectors we obtain equation (4) for the ratio of the resultant to the directly transmitted component,[11,37,39,43]. This ratio provides us with a useful factor for obtaining the directivity of an actual antenna from that of an ideal free space antenna.

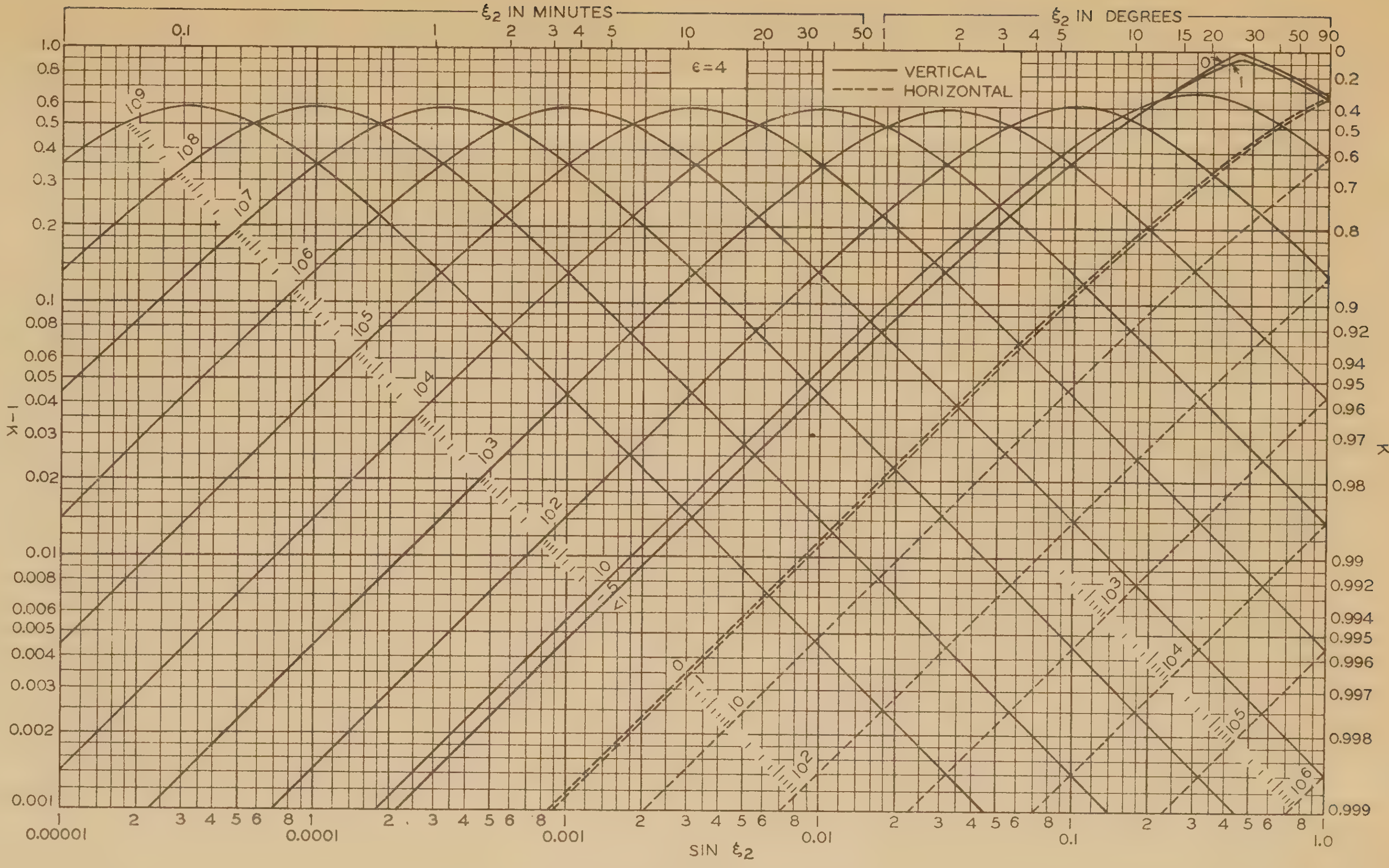


FIG. 8 — MAGNITUDE OF REFLECTION COEFFICIENT FOR $\epsilon = 4$. THE NUMBER ON EACH CURVE GIVES THE VALUE OF $q\left(= 2\sigma/f\right)$ TO WHICH IT APPLIES. VERTICAL POLARIZATION IS SHOWN BY SOLID LINES, HORIZONTAL POLARIZATION BY BROKEN LINES.

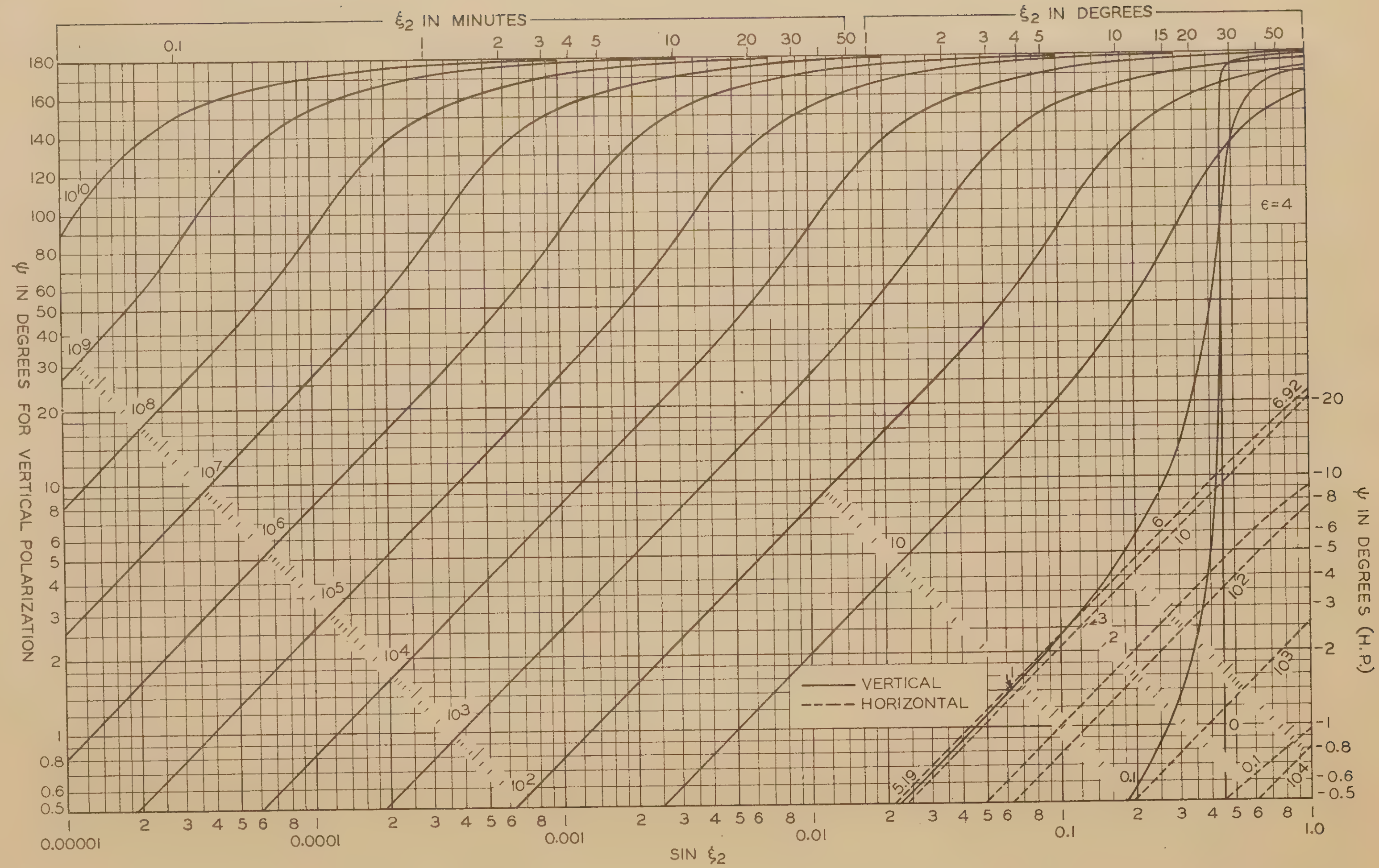


FIG 9 — PHASE SHIFT AT REFLECTION FOR $\epsilon=4$. THE NUMBER ON EACH CURVE GIVES THE VALUE OF $q\,(=2\sigma/f)$ TO WHICH IT APPLIES. VERTICAL POLARIZATION IS SHOWN BY SOLID LINES, HORIZONTAL POLARIZATION BY BROKEN LINES.

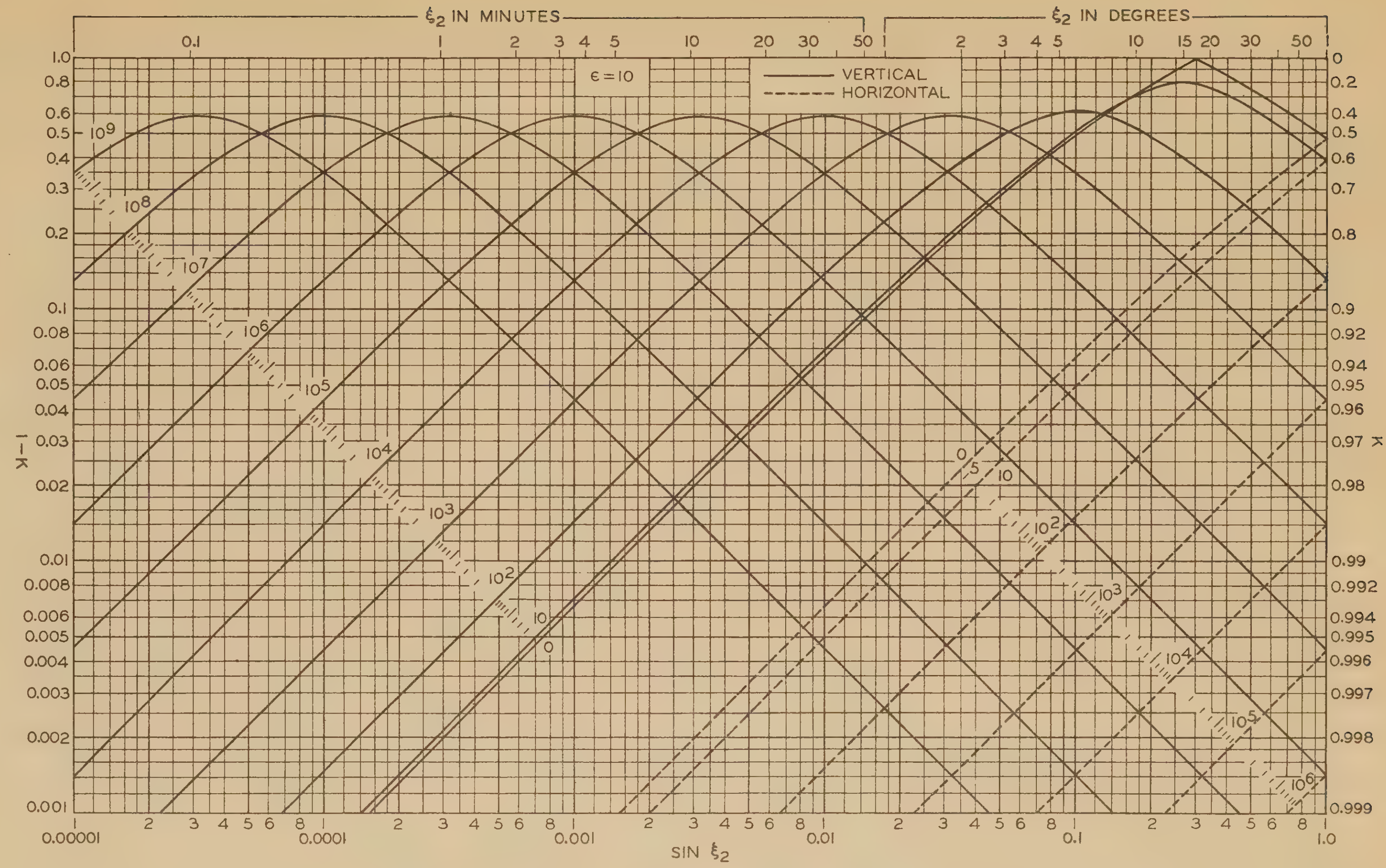


FIG. 10 — MAGNITUDE OF REFLECTION COEFFICIENT FOR $\epsilon=10$. THE NUMBER ON EACH CURVE GIVES THE VALUE OF $q\left(=2\sigma/f\right)$ TO WHICH IT APPLIES. VERTICAL POLARIZATION IS SHOWN BY SOLID LINES, HORIZONTAL POLARIZATION BY BROKEN LINES.

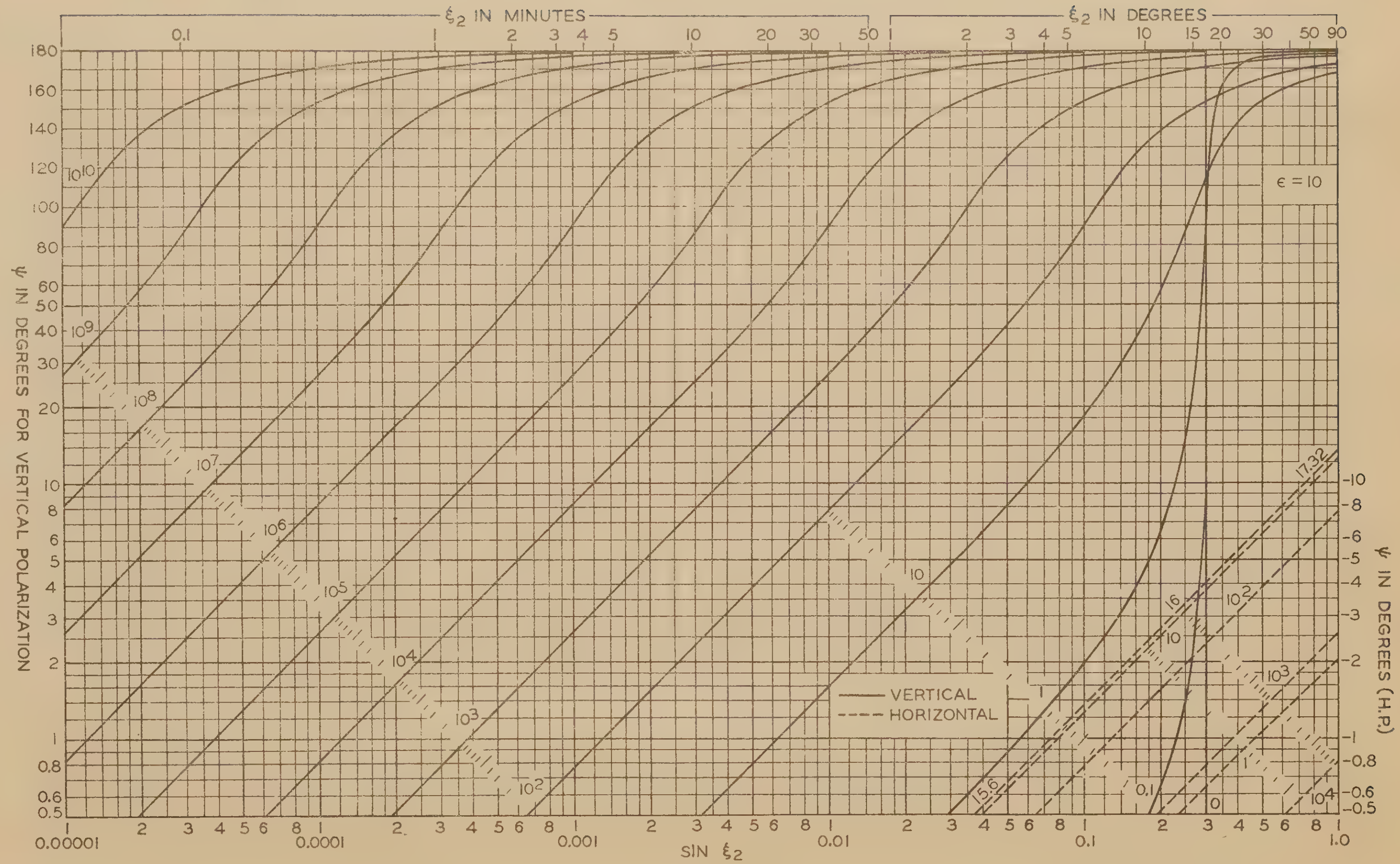


FIG. 11 — PHASE SHIFT AT REFLECTION FOR $\epsilon=10$. THE NUMBER ON EACH CURVE GIVES THE VALUE OF $q\left(=2\sigma/f\right)$ TO WHICH IT APPLIES. VERTICAL POLARIZATION IS SHOWN BY SOLID LINES, HORIZONTAL POLARIZATION BY BROKEN LINES.

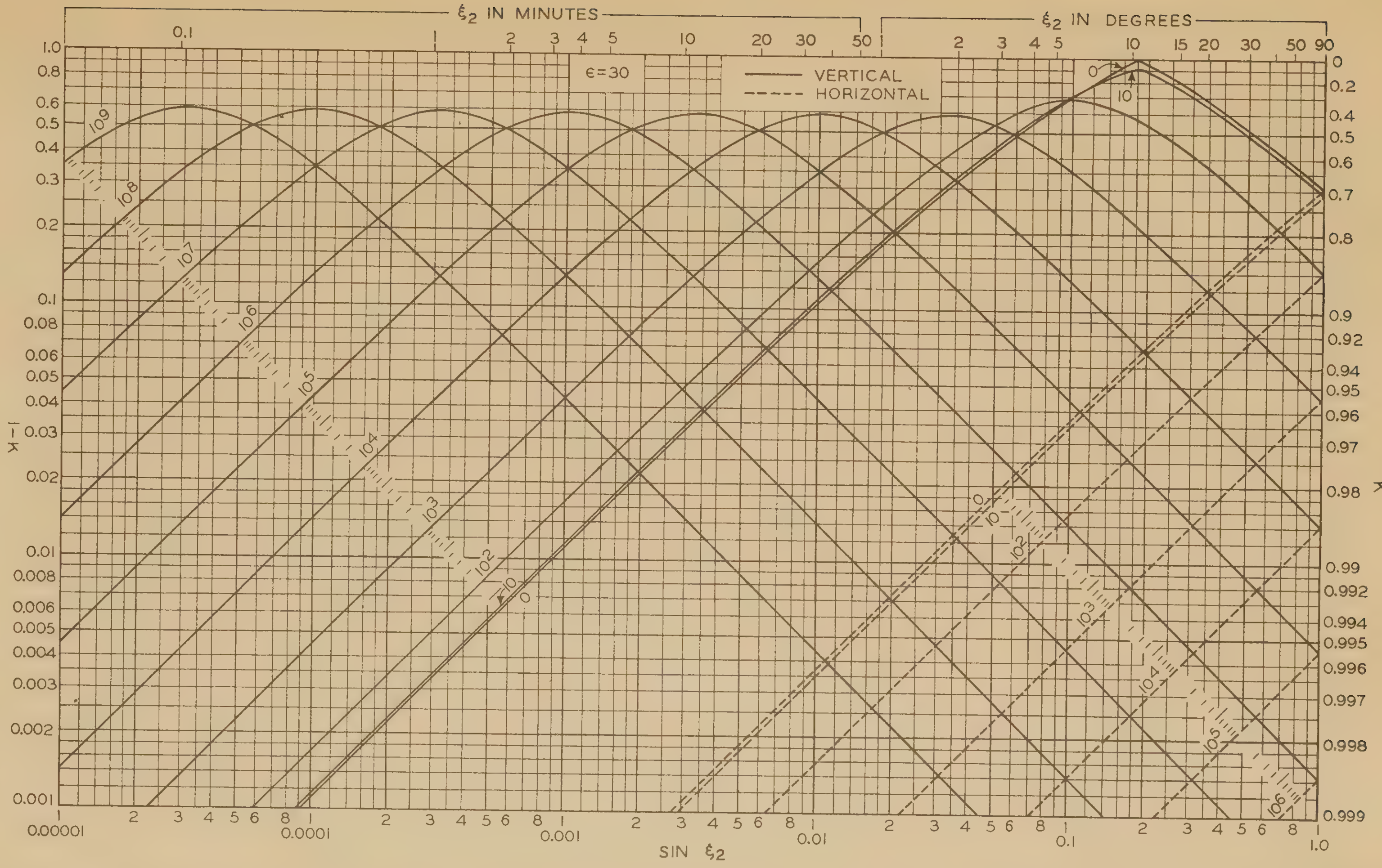


FIG. 12 — MAGNITUDE OF REFLECTION COEFFICIENT FOR $\epsilon = 30$ THE NUMBER ON EACH CURVE GIVES THE VALUE OF $q (= 2\sigma/f)$ TO WHICH IT APPLIES. VERTICAL POLARIZATION IS SHOWN BY SOLID LINES, HORIZONTAL POLARIZATION BY BROKEN LINES.

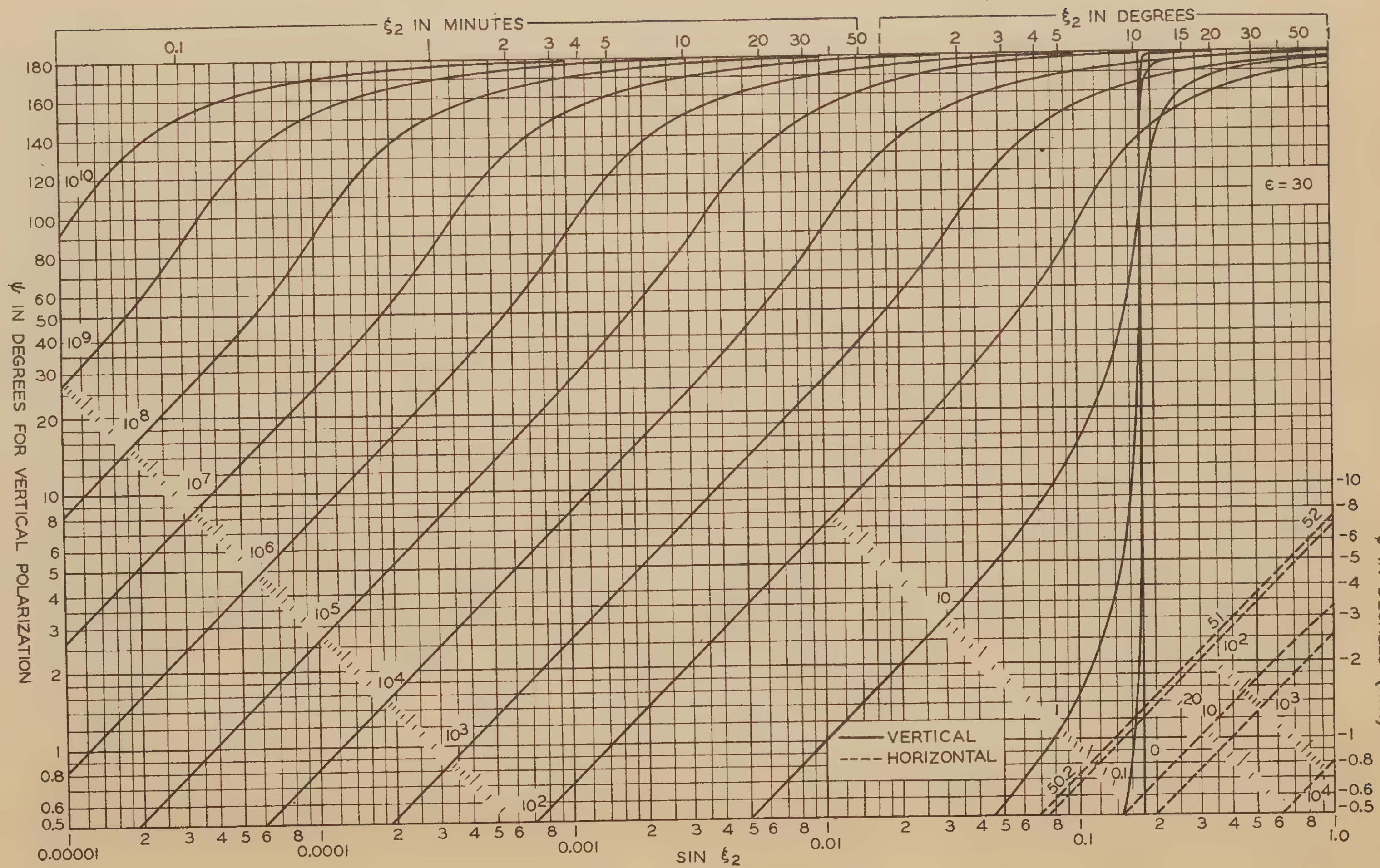


FIG.13 — PHASE SHIFT AT REFLECTION FOR ϵ=30. THE NUMBER ON EACH CURVE GIVES THE VALUE OF $q\,(=2\sigma/f)$ TO WHICH IT APPLIES. VERTICAL POLARIZATION IS SHOWN BY SOLID LINES, HORIZONTAL POLARIZATION BY BROKEN LINES.

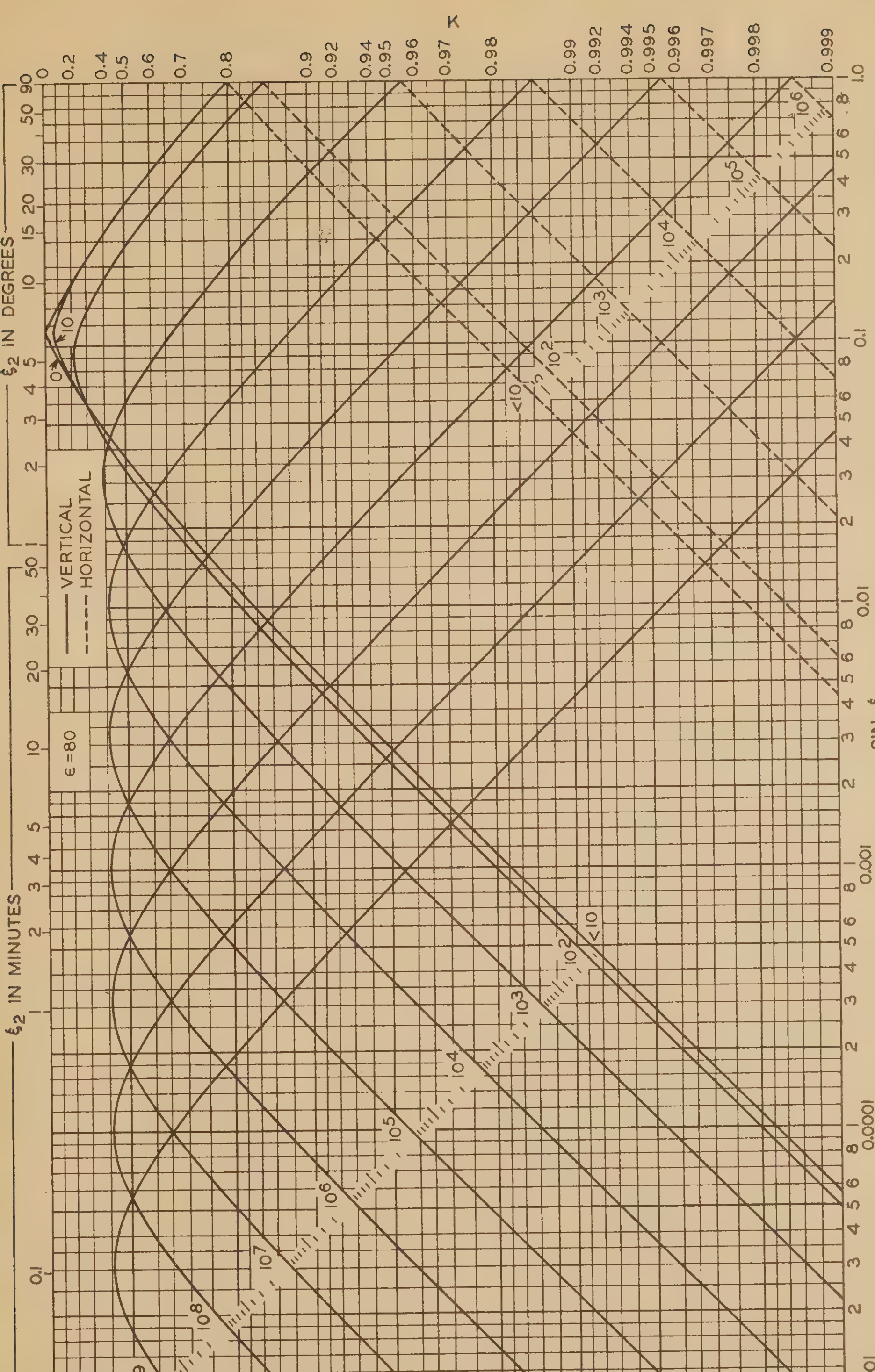


FIG. 14 — MAGNITUDE OF REFLECTION COEFFICIENT FOR $\epsilon = 80$. THE NUMBER ON EACH CURVE GIVES THE VALUE OF $q (= 2\sigma\, f)$ TO WHICH IT APPLIES. VERTICAL POLARIZATION IS SHOWN BY SOLID LINES, HORIZONTAL POLARIZATION BY BROKEN LINES.

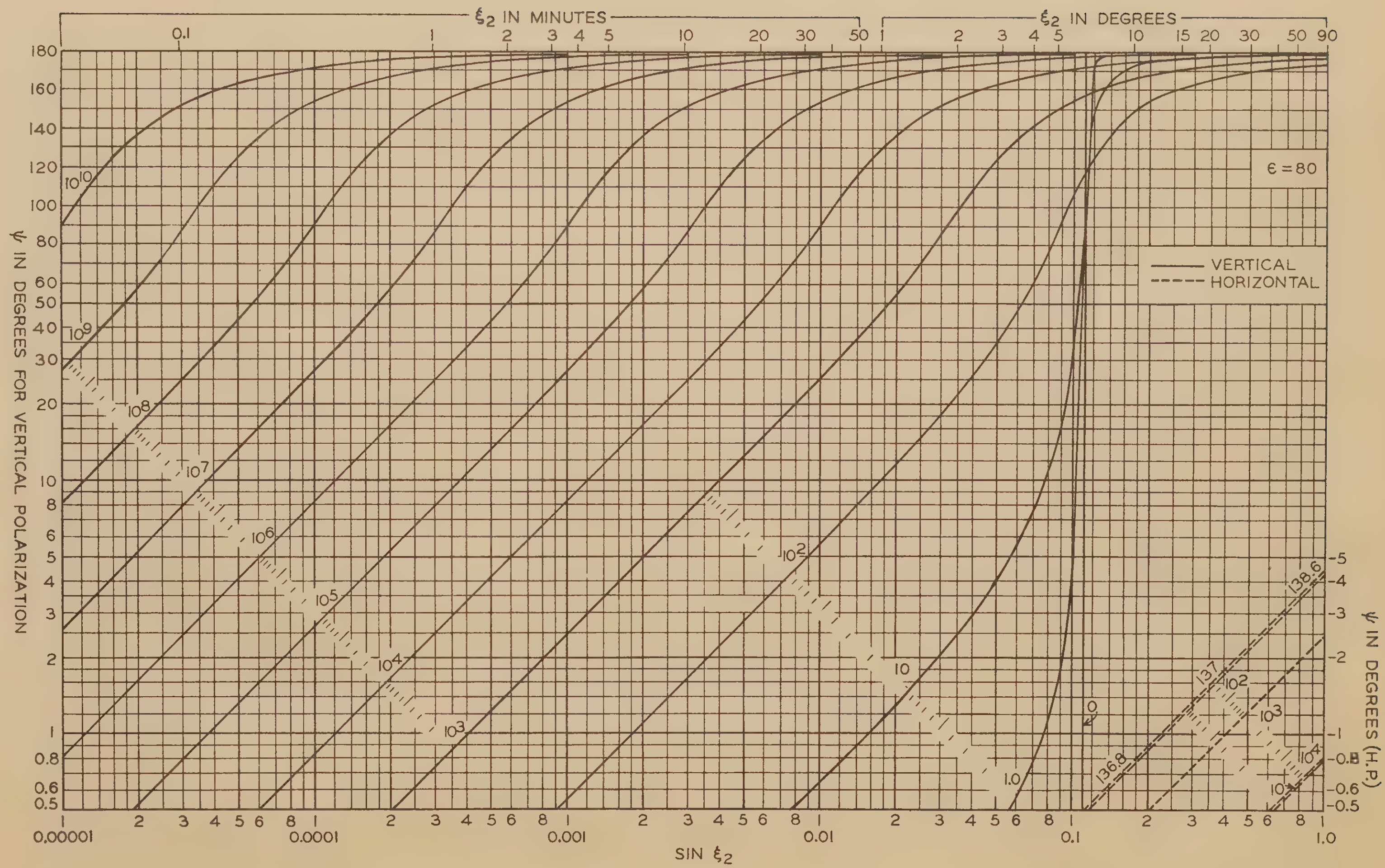


FIG.15 — PHASE SHIFT AT REFLECTION FOR $\epsilon = 80$. THE NUMBER ON EACH CURVE GIVES THE VALUE OF $q\left(=2\sigma/f\right)$ TO WHICH IT APPLIES. VERTICAL POLARIZATION IS SHOWN BY SOLID LINES, HORIZONTAL POLARIZATION BY BROKEN LINES.

$$\frac{E}{E_o} = \left[K^2 + 1 - 2K \cos\left(\psi - \frac{4\pi H}{\lambda} \sin \Delta\right)\right]^{\frac{1}{2}} \tag{4}$$

$$= \left[(1-K)^2 + 4K \sin^2 (\gamma/2)\right]^{\frac{1}{2}}$$

(For nomenclature see Table 1)

where $\gamma = \psi - \frac{4\pi H}{\lambda} \sin \Delta$

and

K = amplitude ratio of reflected to direct wave

$\gamma \pm \pi$ = phase difference between direct and reflected waves

$\psi \pm \pi$ = phase advance at reflection.

Equation (4) gives the ratio of the resultant field strength to the directly transmitted field strength for either plane of polarization, provided that values of K and ψ appropriate for the required plane of polarization are used.

In the application of equation (4) the ratio K or (1-K) and the angle ψ for the specified plane of polarization may be taken from Figures 8 - 15 inclusive, on which these quantities are plotted as functions of the vertical angle ξ_2 (called Δ elsewhere in this discussion), ε and $2\sigma/f$. The dielectric constant ε and the conductivity σ of some commonly encountered types of soil are listed in Table II. Where a soil of high conductivity such as salt marsh is known to exist, the computations for horizontally polarized waves are often simplified by arbitrarily setting $K = 1$; $\psi = 0$, as on Figs. 18 and 21.

To obtain values of 1-K and ψ for smaller values of ξ_2 than are plotted, use is made of the fact that both of these quantities are proportional to ξ_2 in this range of angles,[11]. This linear relationship holds for the lowest

cycle of the curves, so that the parts of the curves in this cycle may be used to obtain values of 1-K and ψ below the edge of the charts as follows. Multiply ξ_2 by the smallest power of ten that will give a value on the chart, read this value and divide by the power of ten originally used. To obtain values of 1-K and ψ for values of $2\sigma/f$ greater than those plotted, divide the given $2\sigma/f$ by some power of one hundred and read the desired value of 1-K or ψ opposite the value of sin ξ_2 multiplied by the square root of the number by which $2\sigma/f$ was divided.

8. Directivity Equations

The power radiated per unit solid angle[38] in a given direction (β,Δ) is called the intensity of radiation Φ, and is given by equation (5) below. This quantity is similar to the power Q passing through a unit area on the surface of a sphere of radius r, and is defined[18] by equation (5A). The field is proportional to the square roots of these quantities.

$$\Phi = \frac{15\pi}{\lambda^2} K_F^2 \tag{5}$$

$$Q = \frac{15\pi}{\lambda^2 r^2} K_F^2 \tag{5A}$$

K_F^2 is a factor which depends upon the coordinates of the direction of radiation, and therefore, determines the antenna directivity. It is expressed in equation (6) in terms of the notation used by Bruce, Beck and Lowry[7], to be consistent with other portions of this discussion. The effect of the conductive ground plane under the antenna will be disregarded for the present and will be considered later in equation (9).

$$(6)\qquad K_F^2 = \left[\frac{32\, I_s^2\, \lambda^2}{\pi^2}\right]\left[\frac{\sin^2 \frac{\ell\pi}{\lambda}(1-\cos\Delta\sin(\phi+\beta))}{(1-\cos\Delta\sin(\phi+\beta)}\right]$$

$$X\left[\cos^2\phi\right] X \left[\frac{\sin^2 \frac{\ell\pi}{\lambda}(1-\cos\Delta\sin(\phi-\beta))}{(1-\cos\Delta\sin(\phi-\beta))}\right]$$

Substituting equation (6) in equation (5) we obtain equation (7) which gives the radiation intensity of a free space rhombic antenna in watts per unit area at a unit distance.

$$(7)\qquad \Phi = \left[\frac{480\,\pi\, I_s^2\, \ell^2}{\lambda^2}\right]\cos^2\phi\left[\frac{\sin^2(\frac{\pi\ell\, K_1}{\lambda})}{\frac{\pi\ell\, K_1}{\lambda}}\cdot\frac{\sin^2(\frac{\pi\ell\, K_2}{\lambda})}{\frac{\pi\ell\, K_2}{\lambda}}\right]$$

where $\left\{\begin{array}{l} K_1 = 1-\cos\Delta\sin(\phi+\beta) \\ K_2 = 1-\cos\Delta\sin(\phi-\beta) \end{array}\right.$

The radiation intensity of equation (7) includes the radiation intensities produced by fields polarized both in a plane parallel to the plane of the antenna and in a plane perpendicular to the plane of the antenna. This resultant may be analyzed into its components as shown in equations (7a) and (7b).

$$(7a)\qquad \Phi = \Phi_H + \Phi_v$$

$$(7b)\qquad \left\{\begin{array}{l} \Phi_H = \dfrac{\Phi\,(\cos\beta - \sin\phi\cos\Delta)^2}{K_1 K_2} \\ \\ \Phi_v = \dfrac{\Phi\,(\sin^2\beta\,\sin^2\Delta)}{K_1 K_2} \end{array}\right.$$

It is apparent that if β or Δ equal zero, Φ_v will vanish, leaving only the component polarized parallel to the plane of the antenna. These effects are more clearly shown in equation (7c) which may be reduced to equation (7) by a simple transformation.

$$\text{(7c)} \qquad \Phi = \left[\frac{480\ \pi^3\ I_s^2\ \ell^4}{\lambda^4}\right] \cos^2 \phi$$

$$\times \left[\frac{\sin\left(\frac{\pi \ell K_1}{\lambda}\right)}{\frac{\pi \ell K_1}{\lambda}}\right]^2 \left[\frac{\sin \frac{\pi \ell K_2}{\lambda}}{\frac{\pi \ell K_2}{\lambda}}\right]^2$$

$$\times \left[(\cos \beta - \sin \phi \cos \Delta)^2 + \sin^2 \beta \sin^2 \Delta\right]$$

Antenna directivity is generally plotted in terms of radiated field or received current, both of which are proportional to the square root of the radiation intensity. The relative directivity, designated as D in the following equations, is therefore a function which completely defines the shape of the directional diagram in terms of field strength. When required the assignment of appropriate values to the constant A will give a result in units appropriate for transmitting or receiving antennas, but ordinarily only relative directivity is of interest.

$$\text{(8)} \qquad D = A \cos \phi \left[\frac{\sin\left(\frac{\pi \ell K_1}{\lambda}\right)}{\frac{\pi \ell K_1}{\lambda}}\right] \left[\frac{\sin \frac{\pi \ell K_2}{\lambda}}{\frac{\pi \ell K_2}{\lambda}}\right]$$

$$\times \left[(\cos \beta - \sin \phi \cos \Delta)^2 + \sin^2 \beta \sin^2 \Delta\right]^{1/2}$$

Since the first and second terms of the final bracket of equation (8) represent respectively the contributions of the two planes of polarization for a free space antenna, the appropriate reflection coefficients must be applied separately when the antenna is erected over an imperfectly conductive ground plane.

$$
(9)\quad \left[\begin{array}{l}
D_H = A \cos \phi \left[\dfrac{\sin \dfrac{\pi \ell K_1}{\lambda}}{\dfrac{\pi \ell K_1}{\lambda}}\right] \left[\dfrac{\sin \dfrac{\pi \ell K_2}{\lambda}}{\dfrac{\pi \ell K_2}{\lambda}}\right] \left[\cos \beta - \sin \phi \cos \Delta \right] X \\[2em]
\qquad \left[K_H^2 + 1 - 2 K_H \cos \left(\psi_H - \dfrac{4\pi H}{\lambda} \sin \Delta\right) \right]^{\frac{1}{2}} \\[3em]
D_V = A \cos \phi \left[\dfrac{\sin \dfrac{\pi \ell K}{\lambda}}{\dfrac{\pi \ell K_1}{\lambda}}\right] \left[\dfrac{\sin \dfrac{\pi \ell K_2}{\lambda}}{\dfrac{\pi \ell K_2}{\lambda}}\right] \left[\sin \beta \quad \sin \Delta \right] X \\[2em]
\qquad \left[K_V^2 + 1 - 2 K_V \cos \left(\psi_V - \dfrac{4\pi H}{\lambda} \sin \Delta\right) \right]
\end{array}\right.
$$

When a perfectly conducting ground is assumed ($K_H = 1$, $\psi_H = 0$; $K_V = 1$, $\psi_V = \pi$), the resultant directivity may be computed by means of equation (9A). For a free space antenna ($K_H = K_V = 0$) substituting equation (9) in (9A) gives us (9B) which may be derived directly from equation (7).

$$D = \sqrt{D_V^2 + D_H^2} \tag{9A}$$

$$D = \frac{I_s \ell \sqrt{480\pi}}{\lambda} \cos\phi \left[\frac{\sin \frac{\pi \ell K_1}{\lambda}}{\sqrt{\frac{\pi \ell K_1}{\lambda}}} \right] \left[\frac{\sin \frac{\pi \ell K_2}{\lambda}}{\sqrt{\frac{\pi \ell K_2}{\lambda}}} \right] \tag{9B}$$

When a ground plane of finite resistivity is assumed, the resultant directivity is no longer exactly defined by equation (9A) on account of the relative phases of the polarization components. However, since the phase and amplitude of the polarization components of a transmitted wave are random, the square root of the sum of their squares represents the most probable value of their resultant.

In the majority of practical applications the directivit of the antenna is specified by its horizontal or vertical plane directivity ($\Delta = 0$, or $\beta = 0$), and for these two special cases $D_V = 0$ and only D_H of equation (9) need be considered. Letting $\beta = 0$ we obtain equation (11) which is the same as that developed by Bruce, Beck & Lowry[7] by a different method of attack.

(11)*

$$D = B \left[\frac{2 \cos \phi}{1 - \sin \phi \cos \Delta} \right]$$

$$\times \left[1 + K_H^2 - 2K_H \cos \left(\psi_H - \frac{4\pi H}{\lambda} \sin \Delta \right) \right]^{\frac{1}{2}}$$

$$\times \left[2 \sin \frac{\pi \ell}{\lambda} (1 - \cos \Delta \sin \phi) \right]^2$$

This equation is useful both for the computation of the vertical directivity and for the development of maximized design equations. If a perfectly conductive ground is assumed equation (12) may be developed by maximizing the factors of equation (11). These functions are plotted on Figure 16.

(12)

$$\left(\begin{array}{l} \phi = \sin^{-1} \cos \Delta \\ \\ \ell = \dfrac{\lambda}{2 \sin^2 \Delta} \\ \\ H = \dfrac{\lambda}{4 \sin \Delta} \end{array} \right.$$

The effect of low angle disturbances may be reduced by means of the design modifications shown in Equation (13). A reduction of the side length suppresses transmission at angles below the maximum and aligns the maximum directly on a preassigned angle Δ. This method is not universally applicable because the alignment is obtained at the expense of approximately 1.5 db in gain,

*Note: For an ideal receiving antenna D represents the current delivered to the terminal load Z_o in microamperes when B is set equal to $E_c\lambda/4\pi Z_o$)

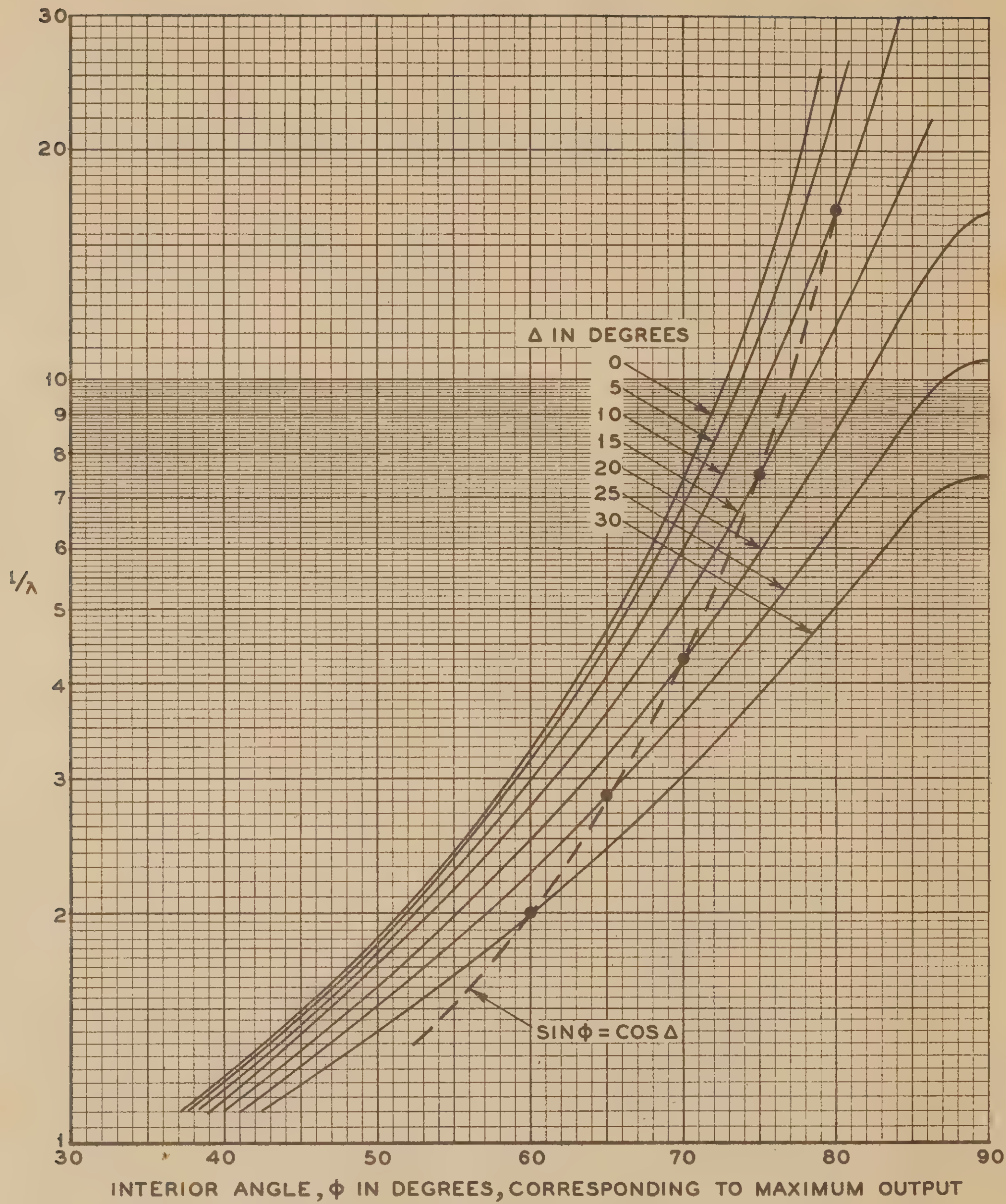


FIG. 16 — FREE SPACE RHOMBIC ANTENNA
MAXIMIZED DESIGN; $\beta = 0$,

$$\tan\left[\frac{\pi l}{\lambda}\left(1-\cos\Delta\sin\phi\right)\right] = \frac{2\pi l}{\lambda}\cos^2\phi\cos\Delta\,\frac{1-\cos\Delta\sin\phi}{\cos\Delta-\sin\phi}$$

OPTIMUM l/λ PLOTTED AS A FUNCTION OF ϕ

$$(13)\qquad \begin{cases} \phi = \sin^{-1} \cos \Delta \\ \ell = \dfrac{0.371\,\lambda}{\sin^2 \Delta} \\ H = \dfrac{\lambda}{4 \sin \Delta} \end{cases}$$

The antenna height and length are both functions of the wavelength and at the lower frequencies it may be found desirable for economic reasons to modify the height with a compensating increase in length or vice versa. Having determined the height at the highest transmission frequency by means of equation (12), a compromise height H' may be assumed and a new length computed by equation (14).

$$(14)\qquad \frac{H'}{\tan\left(\frac{2\pi H'}{\lambda} \sin \Delta\right)} = \frac{\lambda}{2\pi \sin \Delta} - \frac{\ell \sin \Delta}{\tan\left(\frac{\pi \ell}{\lambda} \sin^2 \Delta\right)}$$

If the height, as determined by equation (12) is retained, a reduction of the side length to ℓ' may be compensated by a change in the apex angle to ϕ'.

$$(15)\qquad \phi' = \sin^{-1} \left(\frac{\ell' - 0.371\lambda}{\ell' \cos \Delta}\right)$$

In computing the azimuthal directivity of a horizontal rhombic antenna over a ground plane, if Δ is set equal to zero, the height factor will approximate zero. If Δ is set at higher angles than zero, the effects of vertical polarization will become apparent. Since the azimuthal directivity is not a function of the height or ground constants, it may

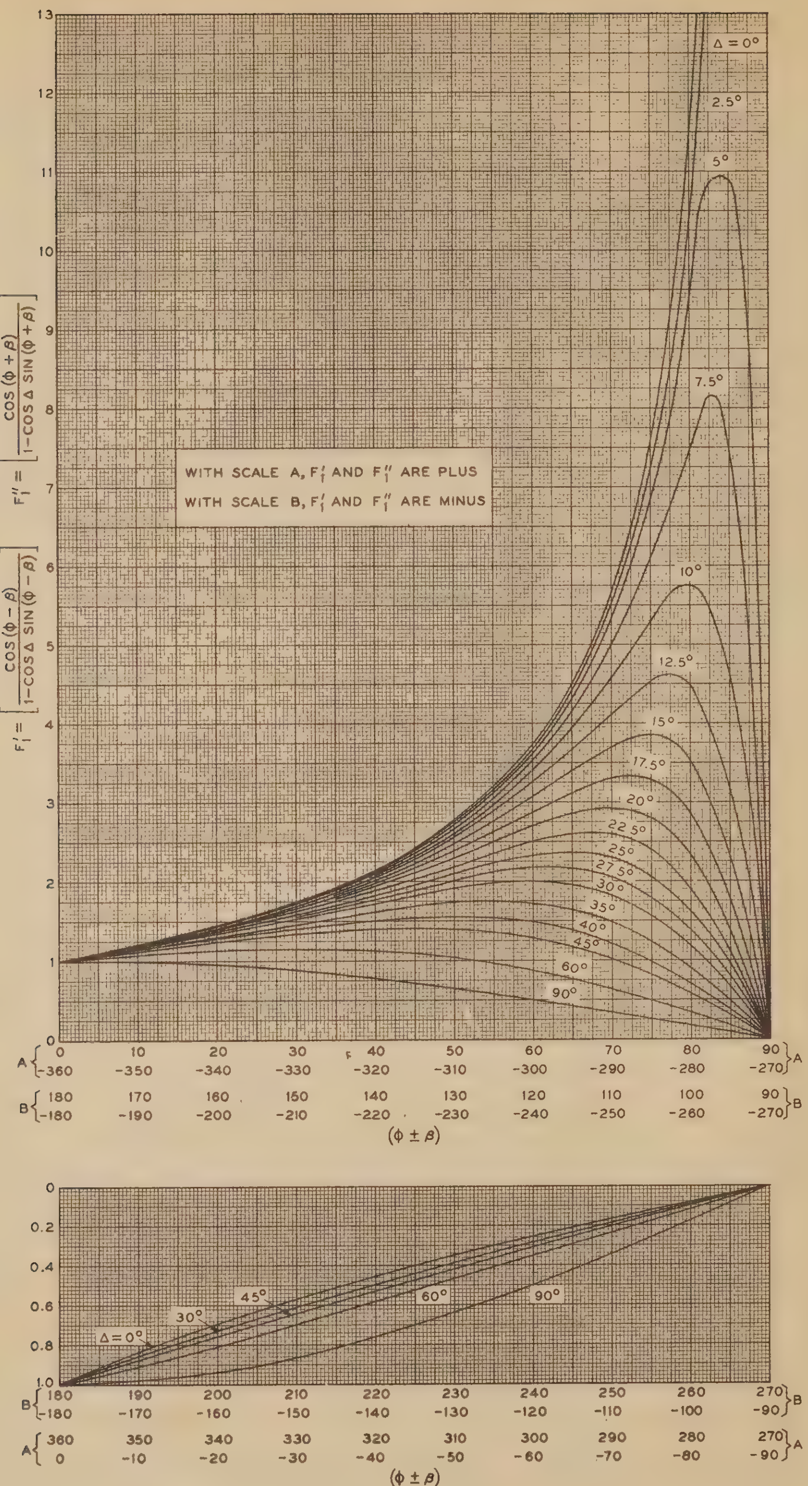


FIG. 17 — FACTOR F_1

$F_1 = F_1' + F_1''$

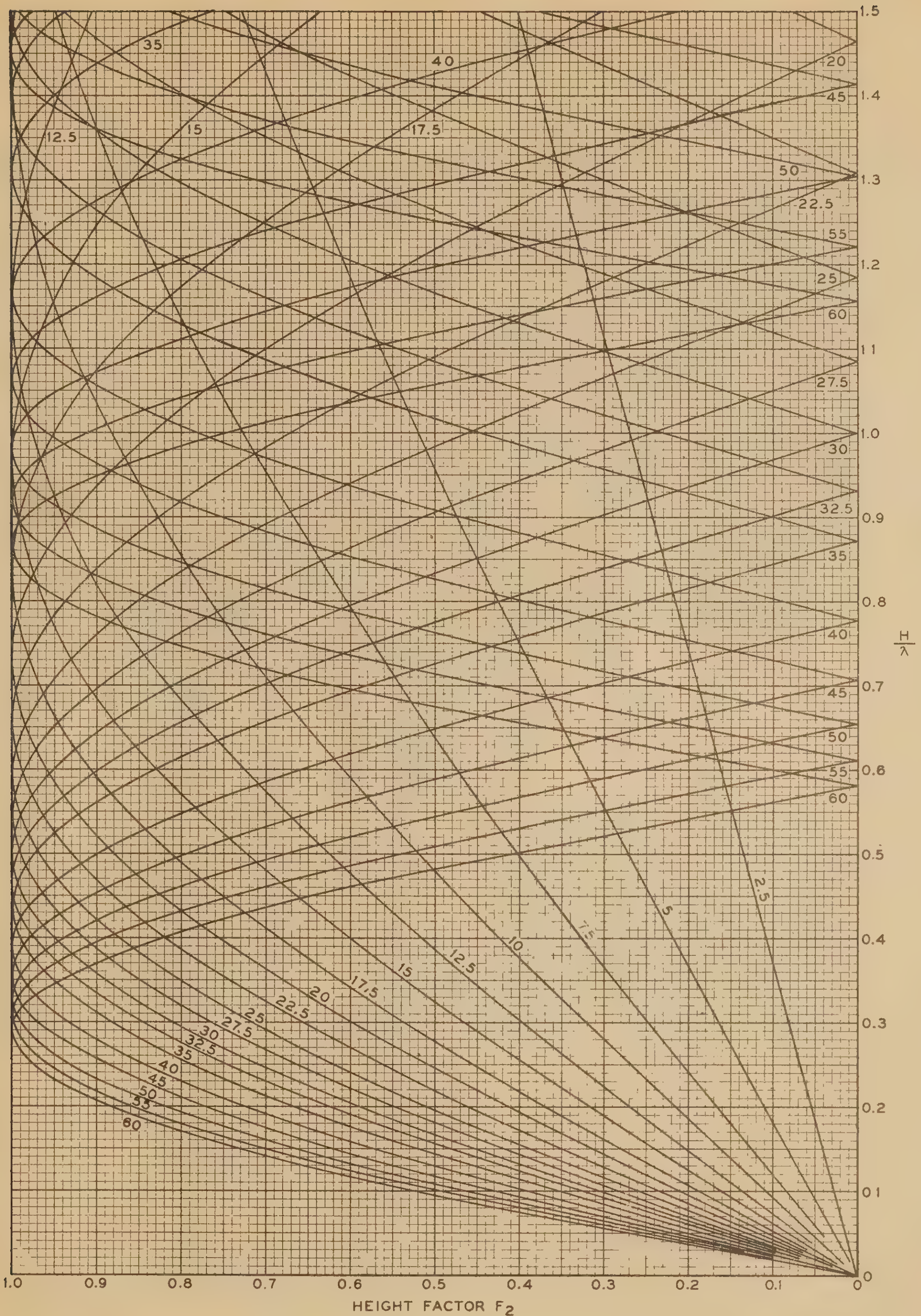


FIG. 18 — HEIGHT FACTOR $F_2 = \text{SIN}\left[\frac{2\pi H}{\lambda}\ \text{SIN}\ \Delta\right]$

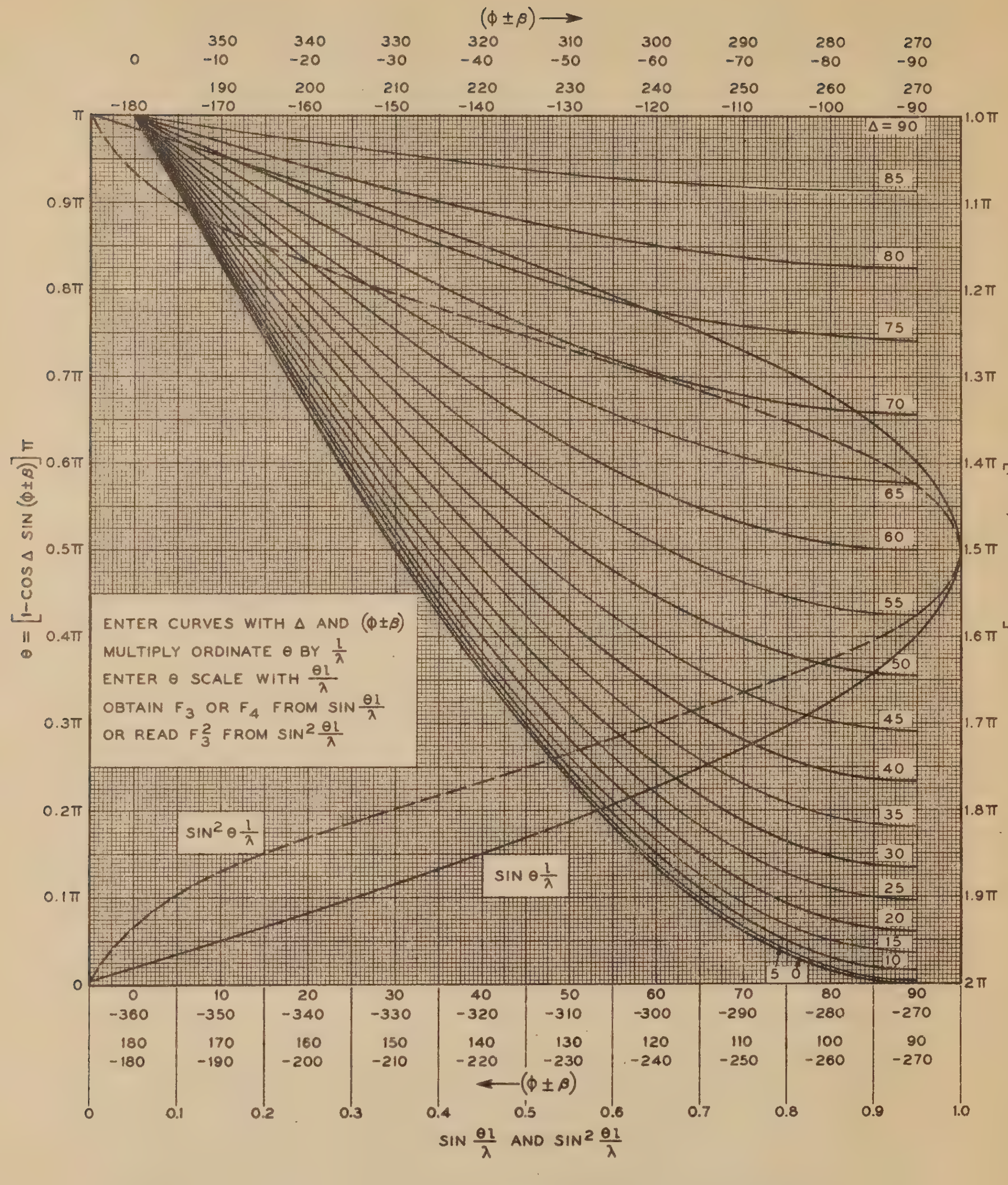


FIG. 19 – FACTOR F_3 AND $F_4 = \sin\frac{l\pi}{\lambda}\left[1 - \cos\Delta \sin(\phi \pm \beta)\right]$

be estimated for antennas whose maxima are at low angles by assuming a free space antenna, letting the ground reflection term of equation (9) or (10) equal unity and Δ equal zero.

In some antenna projects the effect of the vertically polarized component is entirely disregarded and the analysis of the directivity is confined to horizontally polarized waves. This solution is exact when either or both β and Δ equal zero, in which case Φ_V of equation (7b) and D_V of equation (9) become zero, giving equation (10) below when K = 1; $\psi_H = 0$. As Φ_V becomes appreciable relative to Φ_H, the error of neglecting the vertically polarized component becomes larger.

$$(10) \qquad D_H = B\left[\frac{\cos(\varphi-\beta)}{1-\sin(\varphi-\beta)\cos\Delta} + \frac{\cos(\varphi+\beta)}{1-\sin(\varphi+\beta)\cos\Delta}\right]$$

$$\times\ 2\sin\left(\frac{2\pi H}{\lambda}\sin\Delta\right)$$

$$\times\ 2\sin\left[\frac{\pi\ell}{\lambda}(1-\cos\Delta\sin(\varphi-\beta)\right]$$

$$\times\ 2\sin\left[\frac{\pi\ell}{\lambda}(1-\cos\Delta\sin(\varphi+\beta)\right]$$

The directivity factor of equation (10) may be rapid computed by writing it as a product of a constant by four functions as in (10b). The constant may of course be disregarded when only the relative directivity is required.

$$(10b) \qquad D = B\,(F_1)\,(2F_2)\,(2F_3)\,(2F_4)$$

$$= 8B\,F_1\,F_2\,F_3\,F_4$$

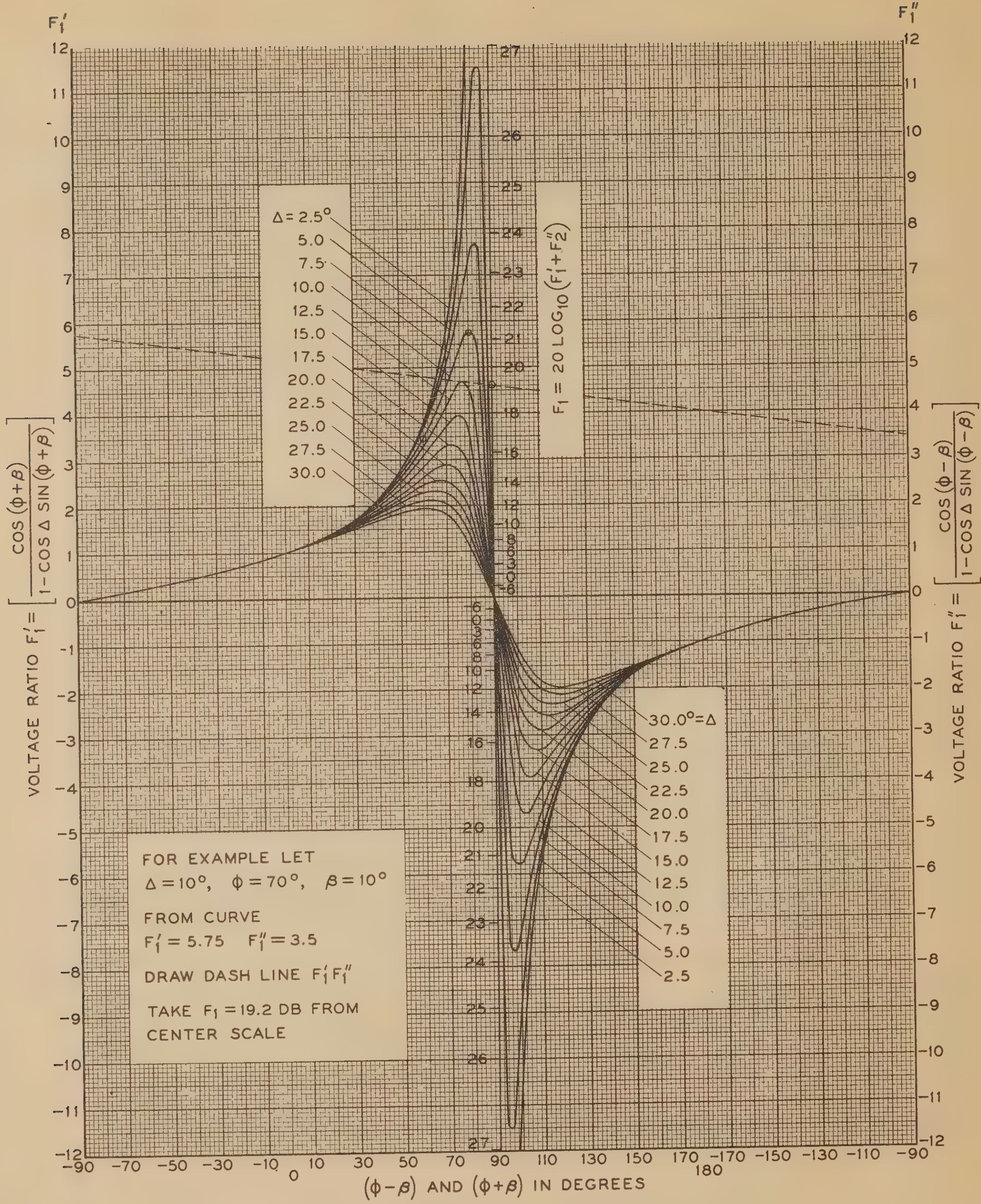


FIG. 20 — F_1 EXPRESSED IN DECIBELS

$$F_{1_{DB}} = 20 \text{ LOG}\left[F_1' + F_1''\right] = 20 \text{ LOG}\left[\frac{\cos(\phi+\beta)}{1-\cos\Delta \sin(\phi+\beta)} + \frac{\cos(\phi-\beta)}{1-\cos\Delta \sin(\phi-\beta)}\right]$$

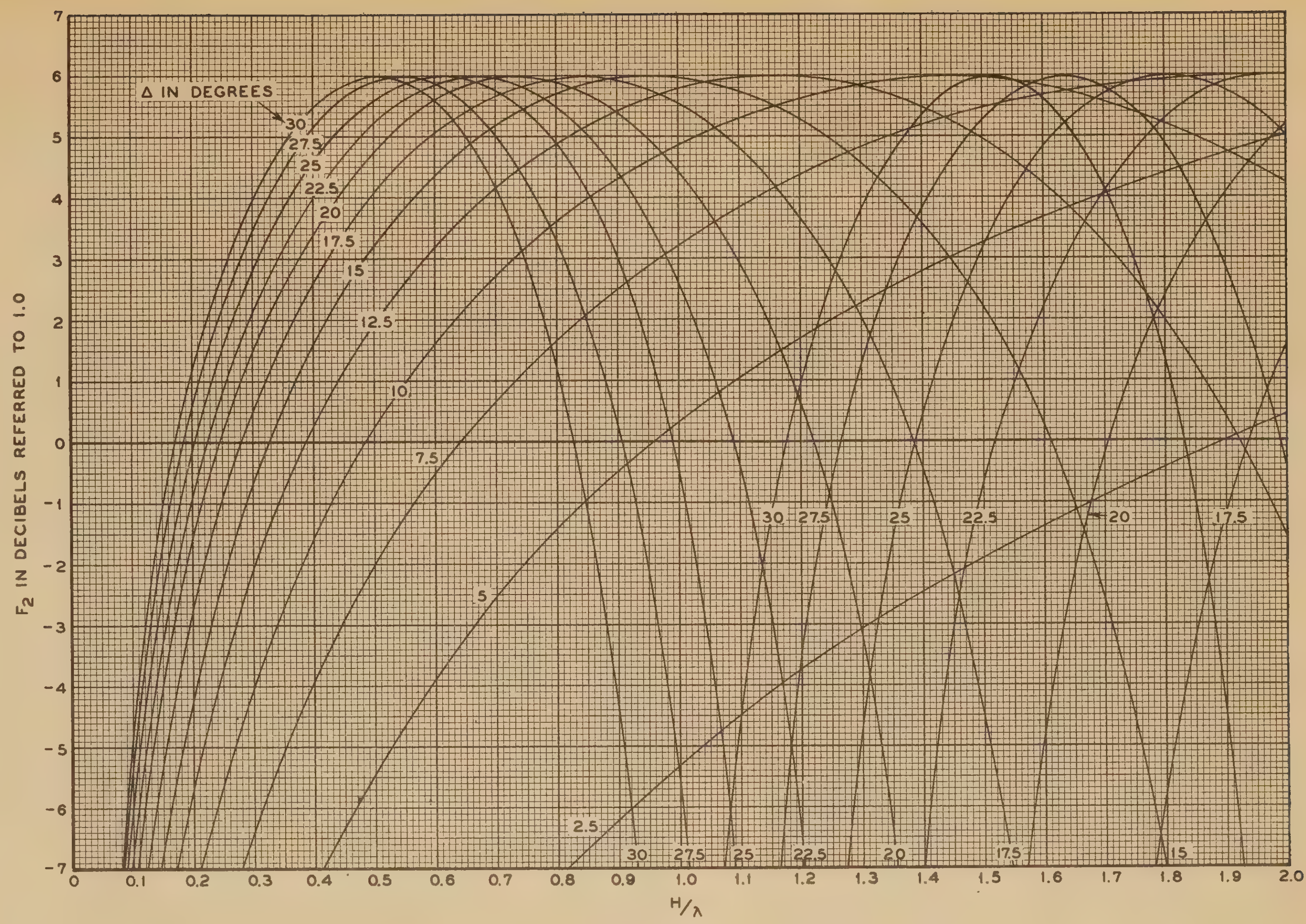


FIG. 21 — F_2 EXPRESSED IN DECIBELS = $20 \log 2 \sin \left[\frac{2\pi H}{\lambda} \sin \Delta \right]$

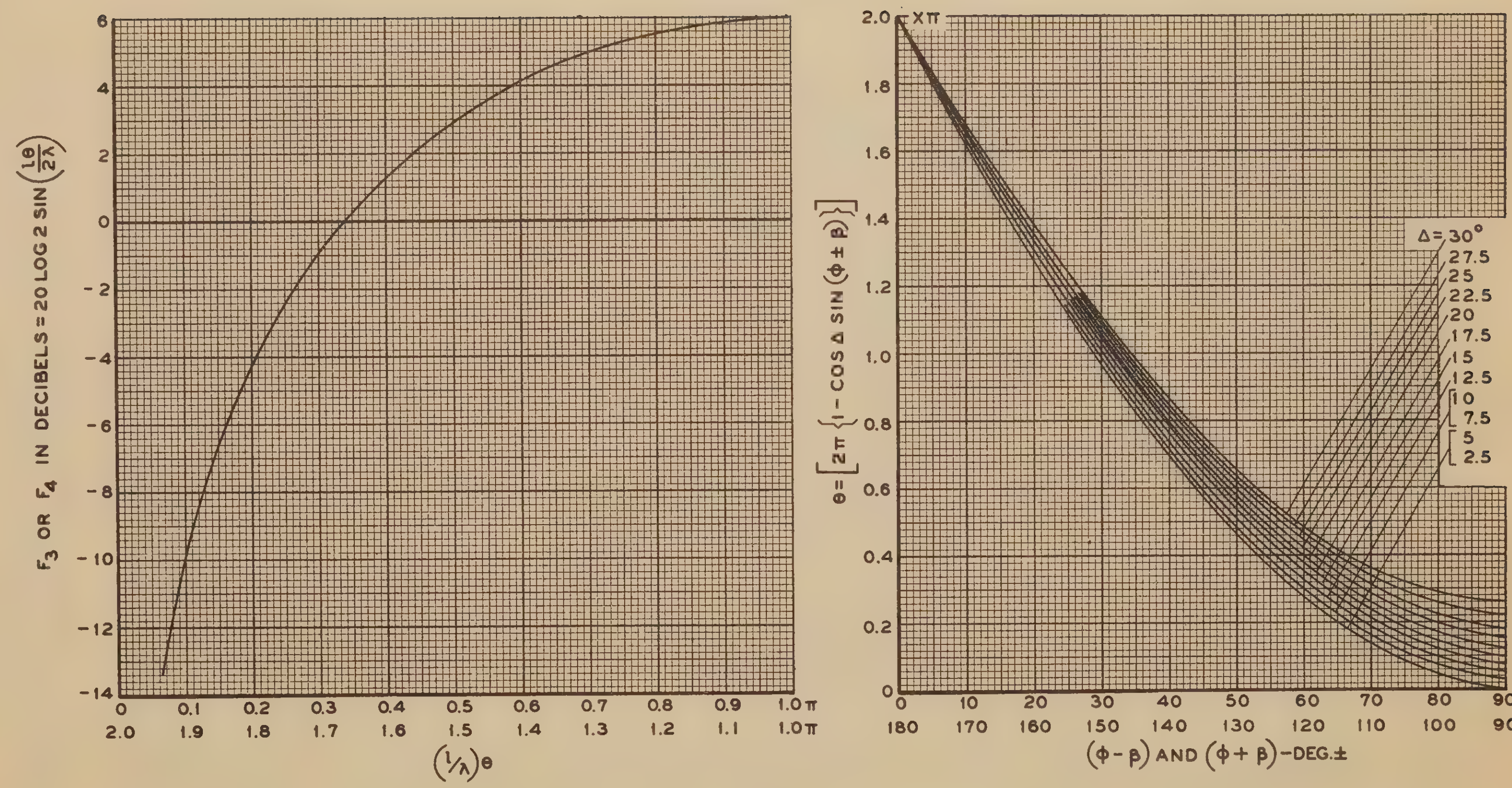


FIG. 22

F_3 IN DB = 20 LOG 2 SIN $\frac{\pi l}{\lambda}\left[1 - \cos\Delta \sin(\phi + \beta)\right]$

F_4 IN DB = 20 LOG 2 SIN $\frac{\pi l}{\lambda}\left[1 - \cos\Delta \sin(\phi - \beta)\right]$

The four functions F_1, F_2, F_3 and F_4 are plotted directly on Figures 17, 18 and 19. In order to expedite computation and make the effect of variations apparent in terms of decibels, Figures 20, 21 and 22 have been plotted in decibels voltage ratio referred to unity. The final directivity factor referred to unity is then the sum of the four decibel values. These decibel ratios are defined in (10c) below.

(10c)
$$F_{1(db)} = 20 \log_{10} (F_1' + F_1'')$$

$$= 20 \log_{10} \left[\frac{\cos(\varphi-\beta)}{1-\cos\Delta \sin(\varphi-\beta)} + \frac{\cos(\varphi+\beta)}{1-\cos\Delta \sin(\varphi+\beta)} \right]$$

$$F_{2(db)} = 20 \log_{10} (F_2)$$

$$= 20 \log_{10} \left[2 \sin\left(\frac{2\pi H}{\lambda}\right) \sin\Delta \right]$$

$$F_{3(db)} = 20 \log_{10} F_3$$

$$= 20 \log_{10} 2 \sin\left(\frac{\pi \ell}{\lambda} (1-\cos\Delta \sin(\varphi-\beta))\right)$$

$$F_{4(db)} = 20 \log_{10} F_4$$

$$= 20 \log_{10} 2 \sin\left(\frac{\pi \ell}{\lambda} (1-\cos\Delta \sin(\varphi+\beta))\right)$$

On Figure 20 the ratios F_1' and F_1'' corresponding to desired values of $(\varphi+\beta)$ or $(\varphi-\beta)$ may be read on the right and left scales of ordinates. By drawing a line between these points as in the example on Figure 20 the decibel ratio corresponding to the sum of ratios F_1' and F_1'', referred to unity may be read from the point where this line crosses the scale of decibels.

Factors $2F_3$ and $2F_4$ are obtained from Figure 22 which is entered with values of $(\varphi+\beta)$ and $(\varphi-\beta)$ respectively Values of θ read on the ordinate scale are multiplied by ℓ/λ and used for entering the abscissa scale at the left side of the plot.

An alternative method for the computation of the directional characteristic is based on Figures 17, 18 and 19. The final result is a numeric obtained by multiplying F_1, F_2, F_3 and F_4 as in equation (10b). F_1 is the sum of two functions, F_1' and F_1'' which are read from Figure 17 corresponding to abscissas $(\varphi+\beta)$ and $(\varphi-\beta)$ and the parameter Δ. F_2 is plotted as a function of H/λ and the parameter Δ, on Figure 18.

F_3 and F_4 are obtained by entering Figure 19 with values of $(\varphi+\beta)$ and $(\varphi-\beta)$ respectively. Read the corresponding values of θ on the scale of ordinates for assigned values of Δ. Then multiply θ by ℓ/λ and enter scale of ordinates with the product $\frac{\ell\theta}{\lambda}$ to obtain $\sin \frac{\ell\theta}{\lambda}$ or $\sin^2 \frac{\ell\theta}{\lambda}$ on the scale of the abscissas.

The final relative radius vector of the directional diagram is the arithmetic product of the plotted factors F_1 F_2 F_3 and F_4. If actual ground constants are used, F_2 must be computed from equation (4) although F_1, F_3 and F_4 may be obtained from the curves.

The reader is again reminded that the use of equation (10), upon which the above two methods depend, may introduce appreciable errors when the direction in question is not included in the planes $\beta = 0$ or $\Delta = 0$. The magnitude of this error is equal to the difference between equations (9) and (10).

For a no loss receiving antenna with a matched load impedance, if the constant B is set equal to $\lambda/4\pi$, the potential measured across the antenna terminals will equal D microvolts when the antenna is excited by a field of one microvolt per meter.

X

CHART AFFECTS OF HGT & L/λ

L TO WAVELENGTH RATIO — (call it increasing leg)

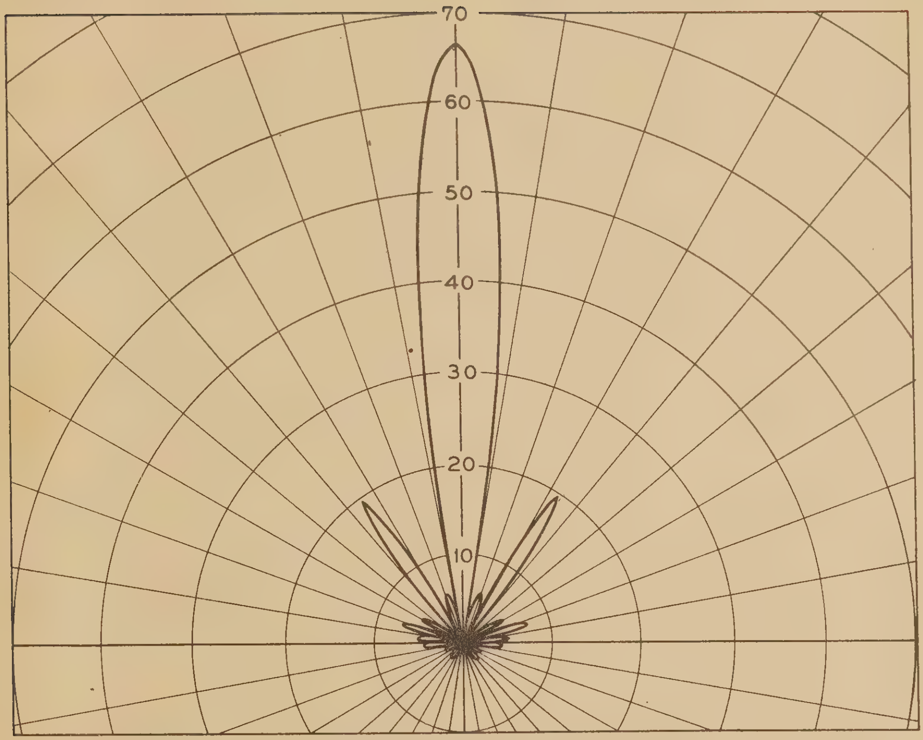


FIG. 23 — AZIMUTHAL DIRECTIVITY OF A RHOMBIC ANTENNA OVER A PERFECTLY CONDUCTING GROUND PLANE WHEN
$\lambda = 16\,m$, $l/\lambda = 6.0$, $\phi = 70°$, $H/\lambda = 1.1$, $\Delta = 10°$

6 λ per leg

$\phi = 70°$

Height = 1.1 λ

Angle above HZ = 10°

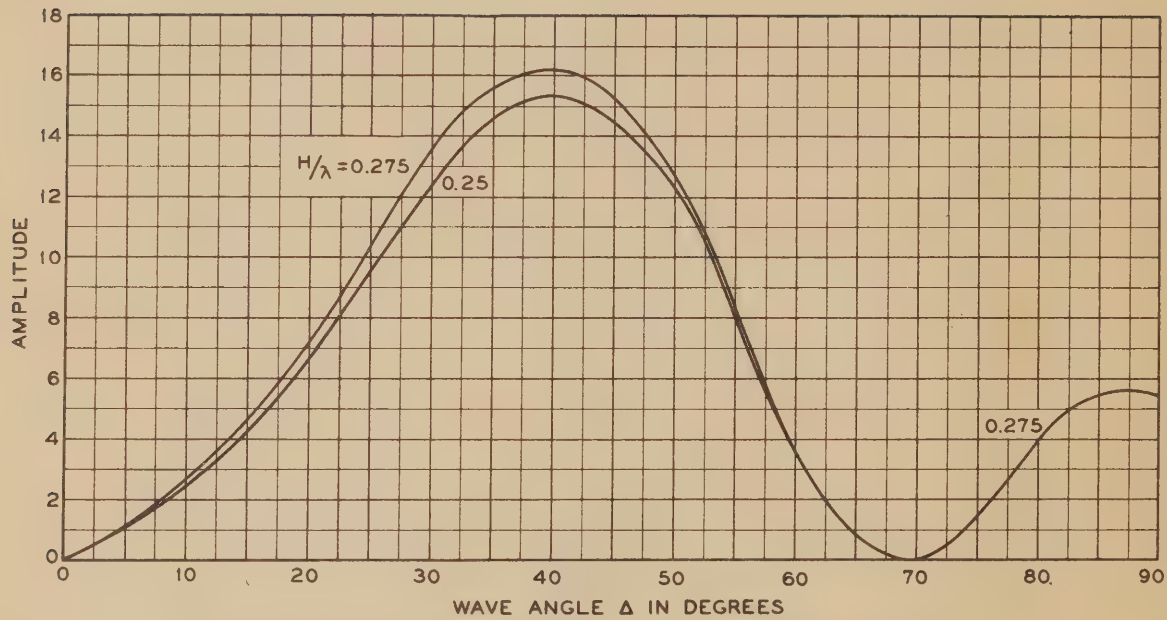


FIG. 24 — VERTICAL PLANE DIRECTIVITY OF A RHOMBIC ANTENNA CONSTRUCTED OVER A PERFECTLY CONDUCTING GROUND PLANE WHEN $\lambda = 64m, l/\lambda = 1.5, \phi = 70^\circ$

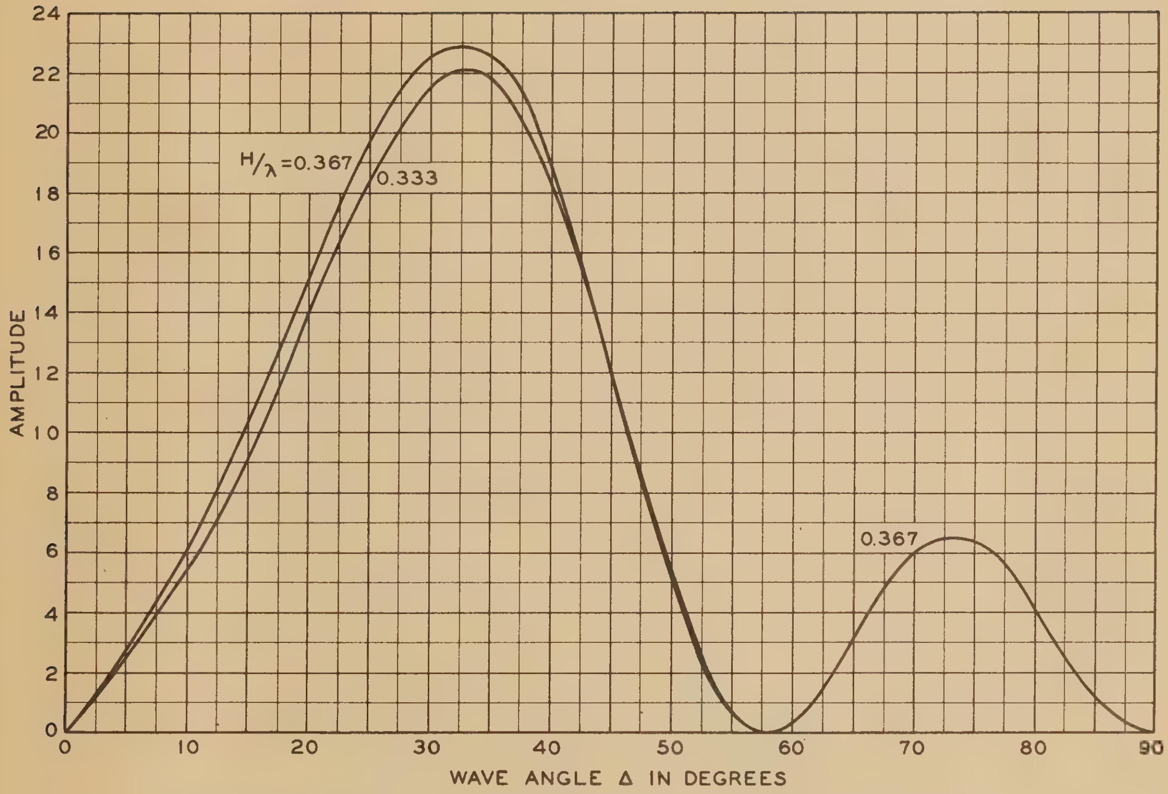


FIG. 25— VERTICAL PLANE DIRECTIVITY OF A RHOMBIC ANTENNA CONSTRUCTED OVER A PERFECTLY CONDUCTING GROUND PLANE WHEN λ=48m $^{l}/_{\lambda}$=2.0 ϕ=70°

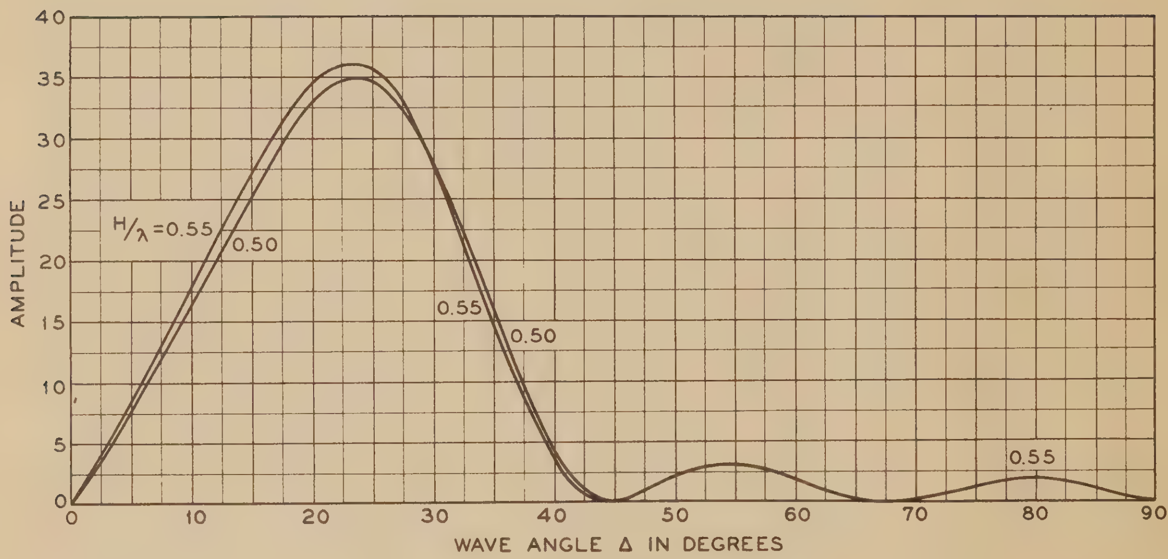


FIG. 26 — VERTICAL PLANE DIRECTIVITY OF A RHOMBIC ANTENNA CONSTRUCTED OVER A PERFECTLY CONDUCTING GROUND PLANE WHEN $\lambda = 32$ m, $l/\lambda = 3.0$, $\phi = 70^\circ$

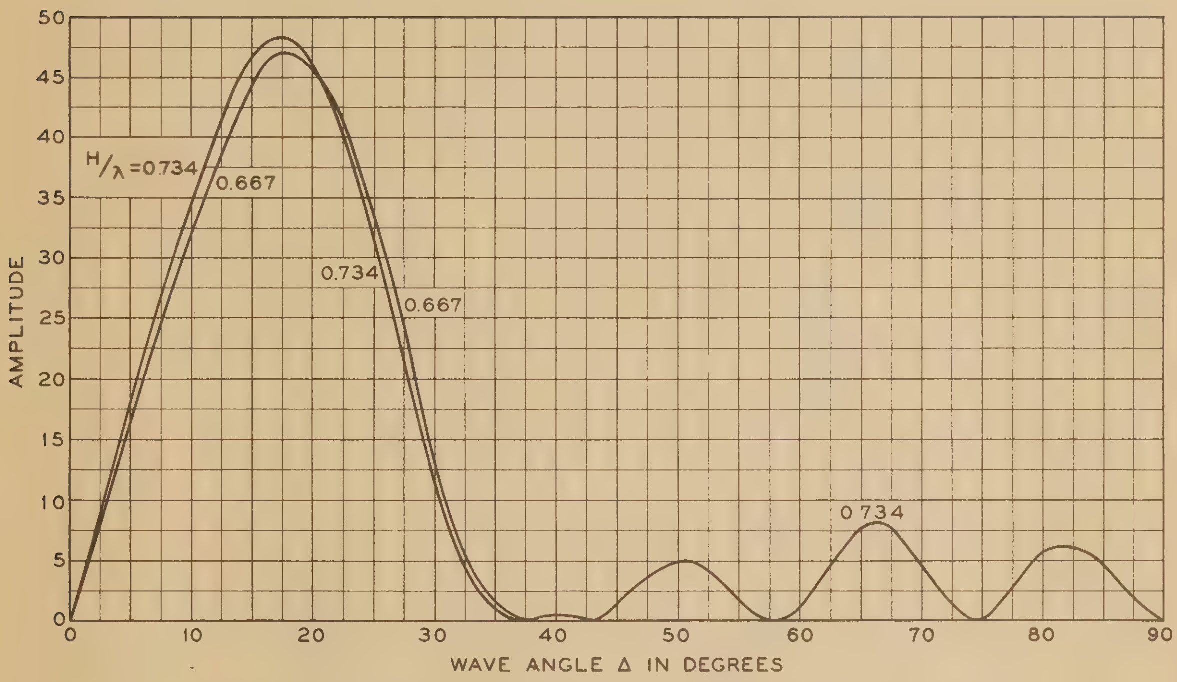


FIG. 27 — VERTICAL PLANE DIRECTIVITY OF A RHOMBIC ANTENNA CONSTRUCTED OVER A PERFECTLY CONDUCTING GROUND PLANE WHEN $\lambda = 24m$, $L/\lambda = 4.0$, $\phi = 70°$

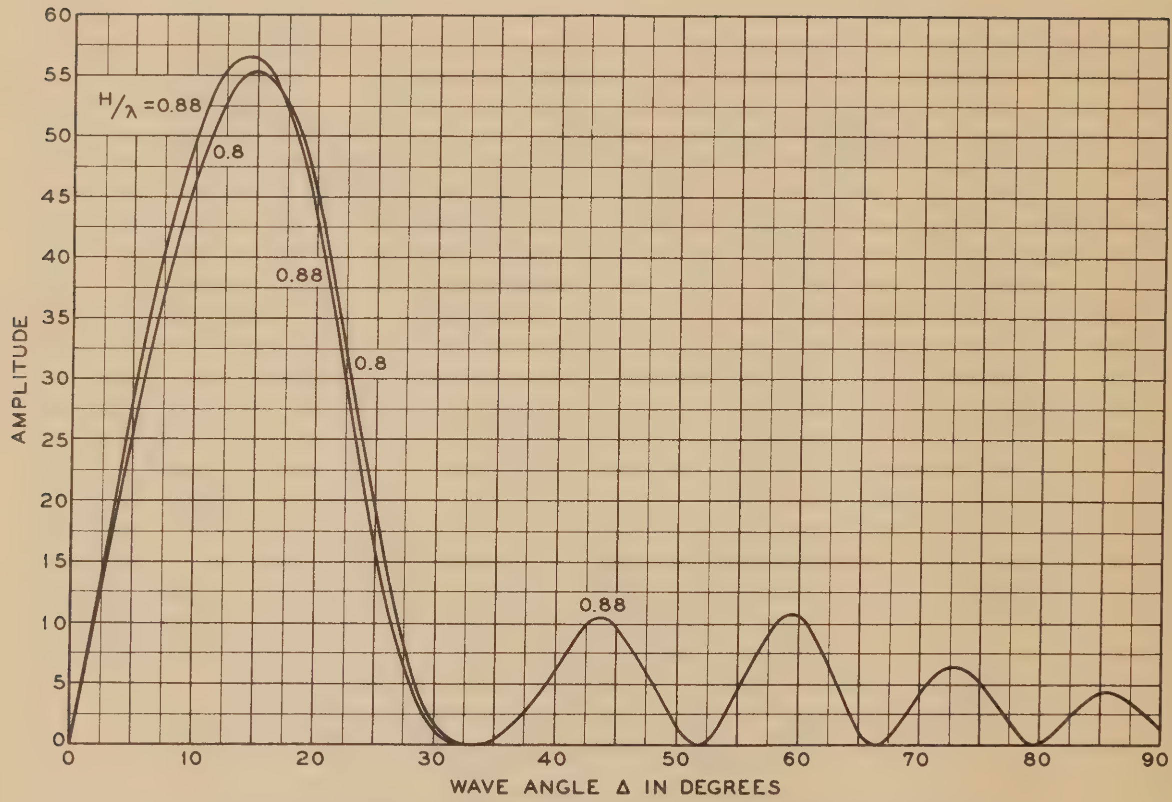


FIG. 28 — VERTICAL PLANE DIRECTIVITY OF A RHOMBIC ANTENNA CONSTRUCTED OVER A PERFECTLY CONDUCTING GROUND PLANE WHEN $\lambda = 20m$, $l/\lambda = 4.8$, $\phi = 70^\circ$

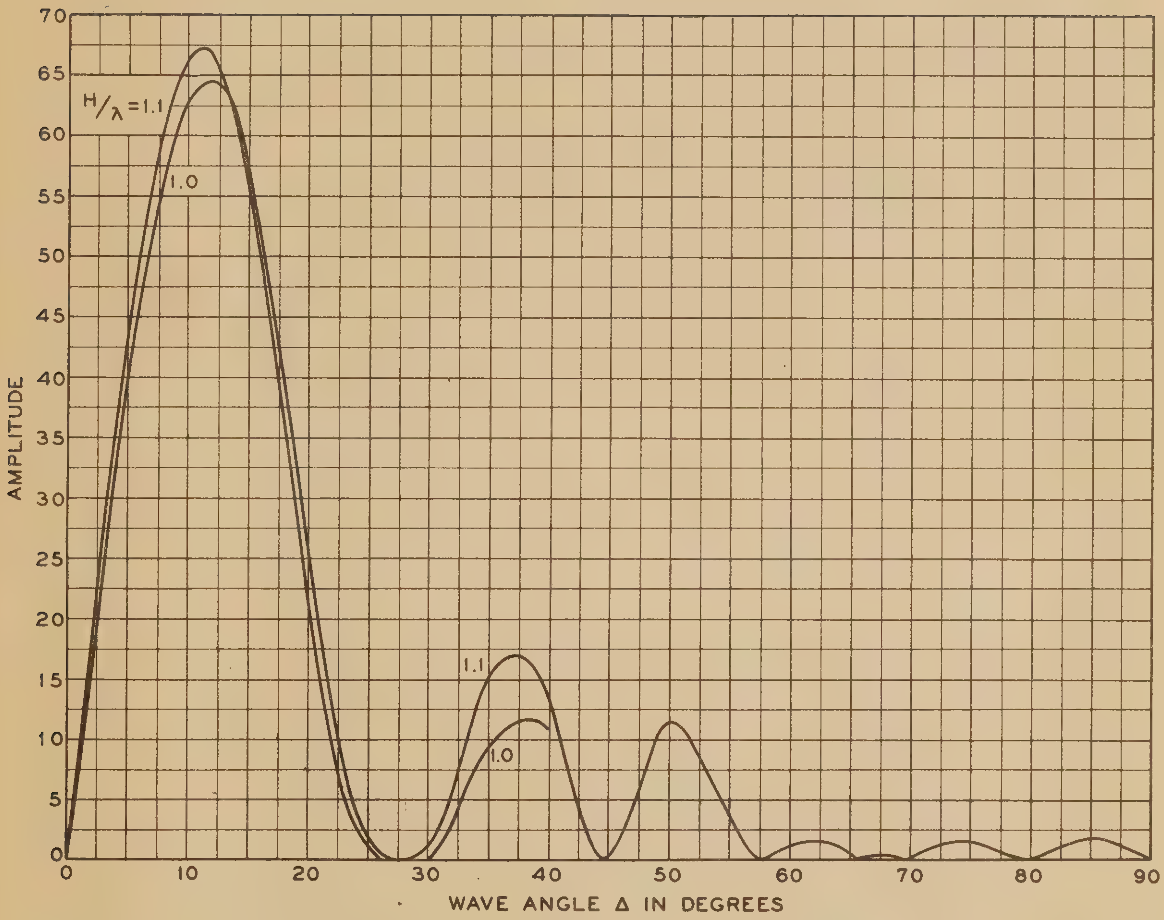


FIG.29 — VERTICAL PLANE DIRECTIVITY OF A RHOMBIC ANTENNA CONSTRUCTED OVER A PERFECTLY CONDUCTING GROUND PLANE WHEN λ=16m, $^l/_\lambda$=6.0, ϕ= 70°

9. Tabulated Functions for Computation

In order to expedite the analytical solution of equations (8), (9), (10) and (11), Tables of sin x/x and 1-cos Δ sin $(\varphi\pm\beta)$ are attached. Table VI gives the values of sin x/x between 0.01 and 15.9 in increments of 0.01 radian, and Table VII gives the values of (1-cos Δ sin $(\varphi\pm\beta)$) in increments of 2°. These tables have been found of service where great accuracy is desired and for checking graphical solutions.

10. Typical Directional Characteristics

It has already been mentioned that the sharpness of the major lobe of the directional characteristics of rhombic antennas intended for transoceanic communication is often intentionally limited by making $\ell \lesssim 6\lambda$ to avoid discrimination against varying angles of arrival or departure. The progressive increases in the broadness of the major lobe and the increasing vertical angle of maximum radiation or reception, in addition to changes in the amplitude of the major and minor lobes, have been plotted on the attached Figures 24 to 29 inclusive for ℓ/λ ratios of 6.0, 4.8, 4.0, 3.0, 2.0, 1.5 for an antenna with 96 meter sides. These graphs are vertical plane directional characteristics obtained by letting $\beta = 0$ at six different transmission frequencies when K = 1 and $\psi = 0$.

11. Measurement and Computation of Antenna Gains

A generalized discussion of antenna gain was presented in the foregoing Section (2), where it was shown that for all types of antenna, the signal gain is the ratio of the powers required to produce equal field intensities at a distant point, through the agency of two transmitting antennas under comparison[4], or the ratio of the powers delivered by two receiving antennas under comparison when excited by the same distant signal.[39] The directivity gain has been defined as the ratio of the power crossing a unit area on the surface

of a sphere of unit radius, in the direction of maximum reception or radiation, to the average power per unit area of the whole sphere. It has been shown on Figure 1 that the signal gain of a directional antenna over an hypothetical non-directional no-loss antenna is equal to the directivity gain minus the ohmic losses in the antenna under test.

This section will be devoted to the more practical aspects of the measurement and computation of the gains of horizontal rhombic antennas. In considering the gains as defined in the above paragraph it at once becomes apparent that the signal gain of an antenna has no absolute meaning unless the properties of the comparison standard to which it is referred are completely and clearly defined. Ordinarily the problem is simplified by comparing the horizontal rhombic antenna under test with a horizontal half wave dipole comparison antenna in the same plane of polarization,[32,39] and at the same distance above the ground. This expedient avoids the necessity of considering two different planes of polarization, two different ground reflection effects, or the effect of height above the ground. In some cases, however, physical difficulties and structural requirements[4] make it more convenient to compare an antenna under test with either an existing array of known characteristics, or with an easily constructed test antenna such as a grounded half or full wave vertical.

It may be well to mention some of the precautions which must be observed in measuring the signal gain. Such measurements on transmitting antennas are usually made by determining the ratio of the field strengths at the receiving terminal of the radio circuit[27] when equal powers are alternately applied to a comparison antenna and to the antenna under test. The signal gain of a receiving antenna is measured by taking the ratio of the powers received on the

comparison antenna and on the antenna under test when excited by the same signal.

Care must be exercised to minimize the effect of stray coupling between the directive antenna under test and the comparison standard antenna which is usually erected nearby. Suppose for example, that an antenna with a signal gain in the neighborhood of 20 db is under test. When the comparison antenna is excited, a coupling between the antennas which will supply energy 20 db below that obtained by direct excitation will be sufficient to make the unexcited directive antenna radiate as strong a field as the excited comparison antenna. The intensity of the resultant field at the distant receiving station will naturally depend upon the relative phases and may have any value between a very small intensity when the two equal components are in phase opposition, and a value 6 db above the field strength which might be expected from a single antenna. Mutual effects of this kind may be greatly reduced by locating the comparison antenna in the 90 degree null sector of the directional antenna, at a distance of at least several wavelengths, if possible. The presence of crosstalk between antennas may of course be eliminated by lowering one of them or rendering it inoperative by sectionalizing its conductors during the period when the other antenna is excited.

In antenna output measurements, such as the determination of transmitting or receiving antenna gain, all radiation phenomena must be confined to the particular antenna in question by minimizing direct radiation from transmission lines. When radiating from a low gain comparison antenna, or when measuring in the null sector of a directional antenna, the low antenna output may become comparable in amplitude with undesired radiation effects of transmission lines. Errors in apparent output due to transmission line effects may be reduced by the obvious methods of shielding, such as the use of low

impedance 2 or 4 wire lines properly balanced to avoid longitudinals, and by the use of coaxial transmission lines. Further reference to spurious transmission line effects will be found in Section 14B.

Both the directive antenna under test and the comparison antenna have directional characteristics which may not at all times coincide with the radio path angles of optimum transmission. For this reason comparisons are generally made by making alternate two minute readings on each antenna over an extended period. The average gain is based on a large number of these series because the instantaneous gain is by no means constant but varies continuously with the hour, season[32], and year, due to the variable nature of the vertical transmission path angle[34].

The evaluation of the exact power expected to be radiated or received in a specified direction by a horizontal rhombic antenna is made difficult by the fact that even if we knew the antenna signal gain of the rhombic referred to a half wave horizontal dipole, the radiation resistance, ground losses, and directivity of the dipole depend upon its elevation above the effective ground plane, and upon the ground constants. The following equations, in terms of the nomenclature of Table I, have been given for the power delivered to a load resistance R at the terminals of a horizontal half wave dipole receiving antenna. Attention is called to the similarity between the bracketed ground reflection terms of equation (4) for the horizontal rhombic and equation (16) for the horizontal half wave dipole comparison antenna[13]. Equations (16), (17) and (18) apply only to the plane of maximum reception, which is normal to the line of the dipole conductors.

$$(16) \qquad P = \frac{E_o^2 \lambda^2}{\pi^2} \left[\frac{R}{(R + R_a)^2} \right] \left[1 - 2K \cos \left(\psi - \frac{4\pi H}{\lambda} \sin \Delta\right) + K^2 \right]$$

Where the load resistance is made equal to the radiation resistance, in order that the maximum power will be delivered to the load, this equation becomes,

$$(17) \qquad P = \frac{E_o^2 \lambda^2}{4\pi^2 R_a} \left[1 - 2K \cos \left(\psi - \frac{4\pi H}{\lambda} \sin \Delta\right) + K^2 \right]$$

If the ground constants approach those of a perfectly conducting earth for which with horizontal polarization $K = 1$ and $\psi = 0$, the above equation is simplified as in the case of the rhombic antenna to equation (18).

$$(18) \qquad P = \frac{E_o^2 \lambda^2}{\pi^2 R_a} \left[\sin^2 \left(\frac{2\pi H}{\lambda} \sin \Delta\right) \right]$$

In cases where the radiation resistance must be matched to the characteristic impedance of an open wire transmission line, the impedance transformation may be accomplished by means of a quarter wave line[4], or by appropriately tapping the line across a portion of the antenna[33]. The radiation resistance of a half wave horizontal dipole in space has been shown[32] to be 73.2 ohms. It also has approximately this value when elevated 0.98λ above a perfectly conducting ground[13] but varies between 82 and 60 ohms as the elevation is changed from 0.9λ to 1.1λ. The terminal impedance of a half wave dipole actually is slightly reactive[33] and may be made a pure resistance by cutting its length from exactly 0.5λ to 0.475λ. The directivity gain of a horizontal half wave dipole in space in a median plane (normal to the wires) is 2.15 db above an hypothetical non-directional antenna.

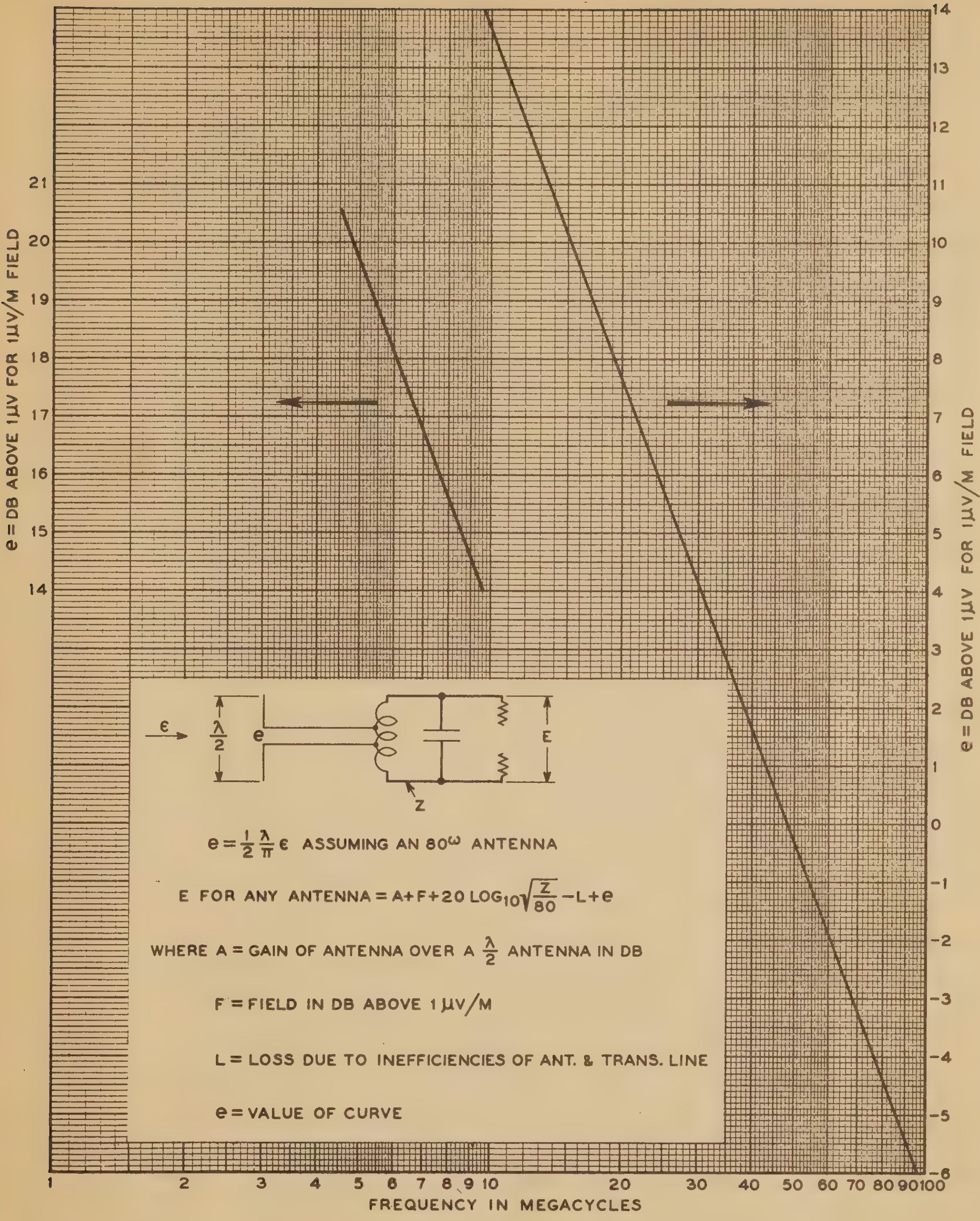


FIG. 30 — VOLTAGE OUTPUT ACROSS 80^{ω} OUTPUT CIRCUIT OF HALF WAVE ANTENNA FOR A FIELD OF 1μV/M

A simplification of the above method of computing the output power or voltage of a half wave dipole antenna may be made by disregarding the effect of the ground on its directive characteristic. If both the rhombic and the comparison dipole are at the same elevation above ground this omission will have little effect on the results of antenna gain measurements (see Figure 32). The absolute value of the voltage or power delivered by a dipole is a function of its elevation, as shown by equations 17 - 18, but the errors introduced by neglecting the component reflected from the ground are often considered negligible relative to the random variations of the radio transmission path. For example if the bracketed term of equation 17 is equated to unity, the expression plotted on Figure 30 may be derived. This drawing is self-explanatory and shows a method for computing the output voltage of a rhombic receiving antenna from comparison with a dipole.[39]

The signal gain of transmitting antennas may likewise be measured by comparison with a half wave dipole at the same height. The power radiated by the dipole[26] may be approximately computed from its free space radiation resistance[32,33,39] by disregarding waves reflected from the ground, as in the method of receiving antenna design explained in the preceding paragraph.

Where it is necessary to use other forms of comparison antenna than the horizontal dipoles described above, the signal gain of these structures must be determined by comparison with reference standard antennas such as dipoles or loops[26].

Assuming that we have measured or computed the potential delivered by the comparison antenna per microvolt field, and have measured the signal gain of the rhombic antenna, these data may be used to compute the signal voltage or power available at the receiver input terminals, from the minimum operating field strength. The power applied to the first circuit of the radio receiver must be well above the irreducible minimum thermal

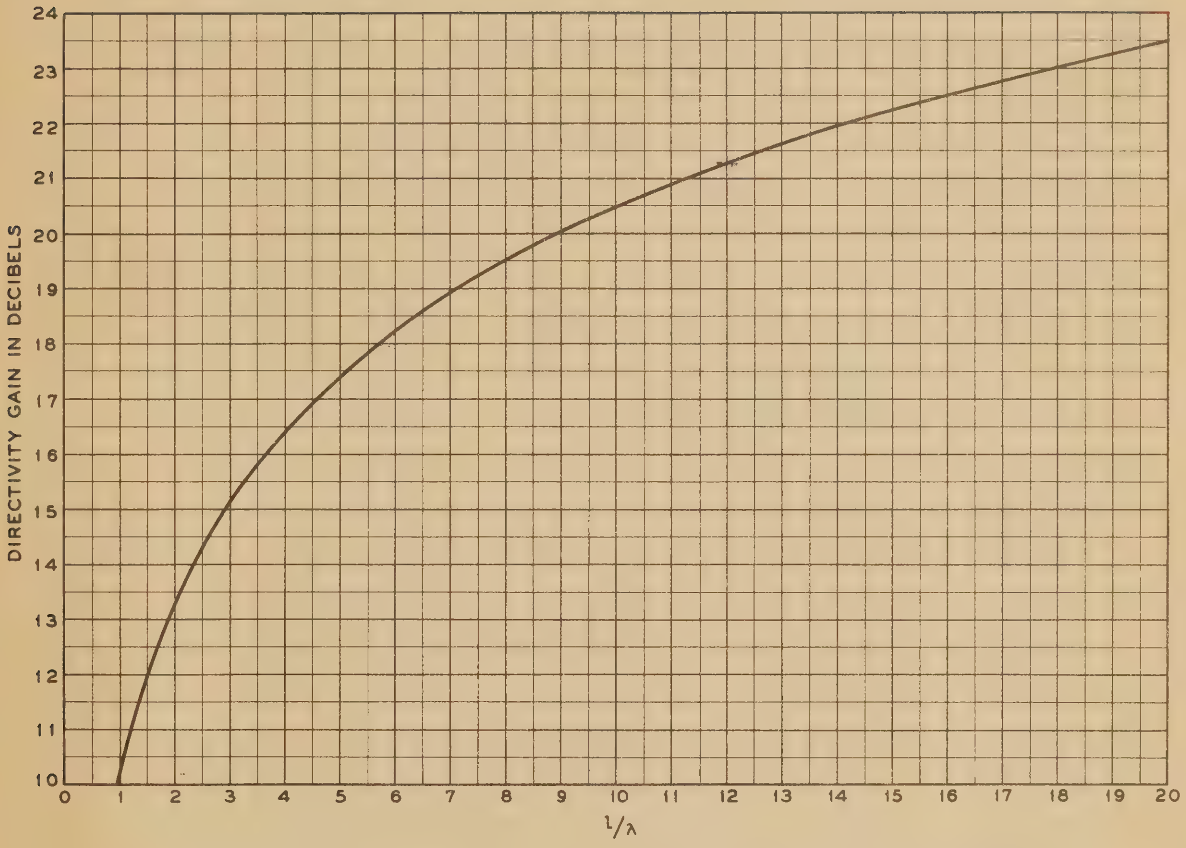


FIG. 31 — FREE SPACE DIRECTIVITY GAIN OF A MAXIMUM DESIGN RHOMBIC,

$\beta = 0,\ \Delta = 0;\ \tan\left[\frac{\pi l}{\lambda}\left(1 - \sin\phi\right)\right] = \frac{2\pi l}{\lambda}\cos^2\phi,$

REFERRED TO AN HYPOTHETICAL OMNIDIRECTIONAL ANTENNA.

(FREE SPACE DIRECTIVITY GAIN OF A HALF WAVE DIPOLE EQUALS 2.15 DECIBELS)

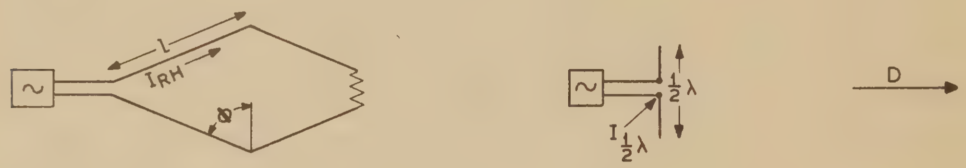
FIG. 32

Effect of Ground on Directivity Gain of Rhombic Over Half Wave Dipole at Same Height. Example Including Gain and Loss Figures for $\ell = 6\lambda$ Rhombic is Given.

SYMBOLS:

I = Antenna Current.
PR = Radiated Power.
RA = Radiation Resistance.
GD = Directivity Gain.
GS = Signal Gain.

Single Prime Sign Refers to Free Space Case.
Double " " " " " Ground ".
Subscript Letters Refer to Type of Antenna.



1. Free Space Case.

I_{RH} and $I_{1/2\lambda}$ adjusted to give equal fields in direction D. Then:

$$\frac{G'D_{RH}}{G'D_{1/2\lambda}} = \frac{P'R_{1/2\lambda}}{P'R_{RH}} = \frac{R'A_{1/2\lambda} \times I^2_{1/2\lambda}}{R'A_{RH} \times I^2_{RH}}$$

2. Ground Case.

I_{RH} and $I_{1/2\lambda}$ as above. The two antennas have identical height factors and give, therefore, for small earth angles equal fields in direction D. Then:

$$\frac{G''D_{RH}}{G''D_{1/2\lambda}} = \frac{P''R_{1/2\lambda}}{P''R_{RH}} = \frac{R''A_{1/2\lambda} \times I^2_{1/2\lambda}}{R''A_{RH} \times I^2_{RH}} = \frac{R''A_{1/2\lambda} \times P'R_{1/2\lambda}/R'A_{1/2\lambda}}{R''A_{RH} \times P'A_{RH}/R'A_{RH}} = \frac{G'D_{RH}}{G'D_{1/2\lambda}} \times \frac{R''A_{1/2\lambda}/R'A_{1/2\lambda}}{R''A_{RH}/R'A_{RH}}$$

EXAMPLE: λ = 16 Meter. ℓ = 96 Meter. φ=70°. Ant. Height = 17.6 Meter. Then:

$R'A_{1/2\lambda} = 73\omega$. $R''A_{1/2\lambda} = 65\omega$ (See Ref. 11)

$R''A_{RH}/R'A_{RH}$ = Appr. 1.1

or $\dfrac{R''A_{1/2\lambda}/R'A_{1/2\lambda}}{R''A_{RH}/R'A_{RH}} = .81 = 0.9$ db

$\dfrac{G'D_{RH}}{G'D_{1/2\lambda}} = 35.0$ (By Mechanical Integration). See also Sec. 11

Then: $\dfrac{G''D_{RH}}{G''D_{1/2\lambda}} = 35.0 \times .81 = 28.3 = 14.5$ db

$\dfrac{G''S_{RH}}{G''S_{1/1\lambda}} = 9.3$ db (Measured)

Then: $\dfrac{G''S_{RH}}{G''S_{1/2\lambda}} = 11.0$ db (taking $\dfrac{G'S_{1/1\lambda}}{G'S_{1/2\lambda}} = 1.7$ db)

The heat loss in the half wave dipole is negligible. The total heat loss in the rhombic is then 14.5 - 11.0 = 3.5 db.

Rhombic heat loss due to ground = .7 db.
Rhombic heat loss in termination resistance is then 2.8 db

agitation noise, which is 43.8 db below 1 micromicrowatt for a 5,000 cycle transmission band[16]. Allowing for a minimum signal-to-noise ratio of 15 db and assuming that other apparatus and circuit noises have been reduced to a level where their interfering effect is negligible, the minimum useful signal power delivered by an antenna is in the neighborhood of 28.8 db below 1 micromicrowatt for a 5,000 cycle receiver band width.

The directivity gain of a rhombic antenna operating over a ground of normal characteristics is rather difficult to compute analytically, although it has been closely approximated by mechanical integration. The directivity gain of a maximum output rhombic in free space is plotted on Figure 31 and the relationship between this gain and the results of measurements on actual antennas is shown on Figure 32. The directivity gain as shown on Figure 31 is referred to an hypothetical non-directional antenna of zero gain. As previously mentioned, in the direction of optimum transmission the horizontal half wave dipole has a gain of 2.15 db in space. The main advantage of using a horizontal dipole for a comparison antenna is the fact that when it is constructed in the same plane as the rhombic, the effect of wave polarization and ground reflections may be disregarded since they are substantially the same in each case. The directivity gain of the rhombic above the horizontal dipole therefore is the inherent space gain of the particular rhombic minus the 2.15 space gain of the horizontal dipole.

Summarizing the above discussion, we may compute the directivity gain, G_d, of a horizontal rhombic antenna of maximized design relative to a horizontal half wave dipole at the same height, by means of equation (19) below where G_D is obtained from Figure 31.

$$G_d = G_D - 2.15 \tag{19}$$

The signal gain of a horizontal rhombic of maximized design erected over a partially conductive ground plane, relative to a horizontal half wave dipole at the same height, may be roughly approximated by means of equation (20) below. The derivation of this relation is shown on Figure 32.

$$
\begin{aligned}
G_s &= G_D - 2.15 - 0.9 - 3.5 \\
&= G_D - 6.5
\end{aligned}
\qquad (20)
$$

Since the above relations are true only for maximized design antennas as defined by equation (12), they become increasingly inaccurate as the operating frequency departs from that for which the optimum design parameters were determined.

12. Multiple Wire and Twin Types

Improved operation may be obtained by means of modified types of construction such as multiple wire antennas and twin antennas. Multiple wire antennas such as shown on Figure 43 and described in more detail in Section 15A, were initially suggested to minimize impedance variations with frequency, and to secure a low and uniform 600 ohm antenna terminal impedance to match an open wire transmission line without the necessity of a coupling transducer. Twin antennas consist of two identical rhombic antennas in broadside array. The members of a pair need not be separated more than a few feet between adjacent corners and in fact a common pole is generally used to support the midpoint of the system. The twins may be connected to the apparatus by means of two independent transmission lines of the same electrical length, which are multipled within the building, thereby permitting the independent use of the antennas in emergencies.

It has been reported that three wire single rhombic antennas not only have desirable impedance properties, but also may have higher signal gains and a freedom from precipitation static when used for receiving. Comparative tests under ordinary operating conditions on a three wire rhombic and an adjacent similar single wire rhombic, seem to show signal gains of 1.5 db

at 19 mc. and 0.7 db at 9 mc. in favor of the multiple wire antenna. This apparent improvement is probably due to a lower propagation constant along the antenna conductors, arising from a lower resistance and more uniformly distributed capacity.

Similar direct comparisons between the signal-to-noise ratio obtainable with single and multiple wire rhombic antennas showed that during periods of precipitation static a mean reduction in noise of 9 db was apparently obtained on the multiple wire type. Although precipitation static occurs during only a small percentage of the total operating time, in most localities the small added expense of this type of construction may be worth while.

Two rhombic transmitting antennas have been fed in parallel to obtain an increase in signal gain of approximately 3 db. Comparative tests between one and a combination of both members of such an identical pair of rhombic antennas for receiving, showed improvements of the order of 2 db in signal-to-noise ratio.

13. Measurement and Significance of Terminal Impedances

Optimum performance of the complete system may only be secured when all circuit impedance irregularities have been minimized and the antenna has been properly terminated at one end on the coupling network or transmission line, and at the other end in its resistance termination. The terminal impedances of all elements of the antenna system are usually separately checked over the whole operating frequency range. In general it is convenient to thoroughly check the transmission and impedances of the antenna coupling network in the laboratory. The impedances of the transmission line and the antenna are of course measured after installation, when terminated in impedances which closely simulate the values actually to be used. Composite impedance measurements over a wide range of frequency are often made of the coupling unit when working into the properly terminated antenna, and of the complete system, from the apparatus end of the transmission line.

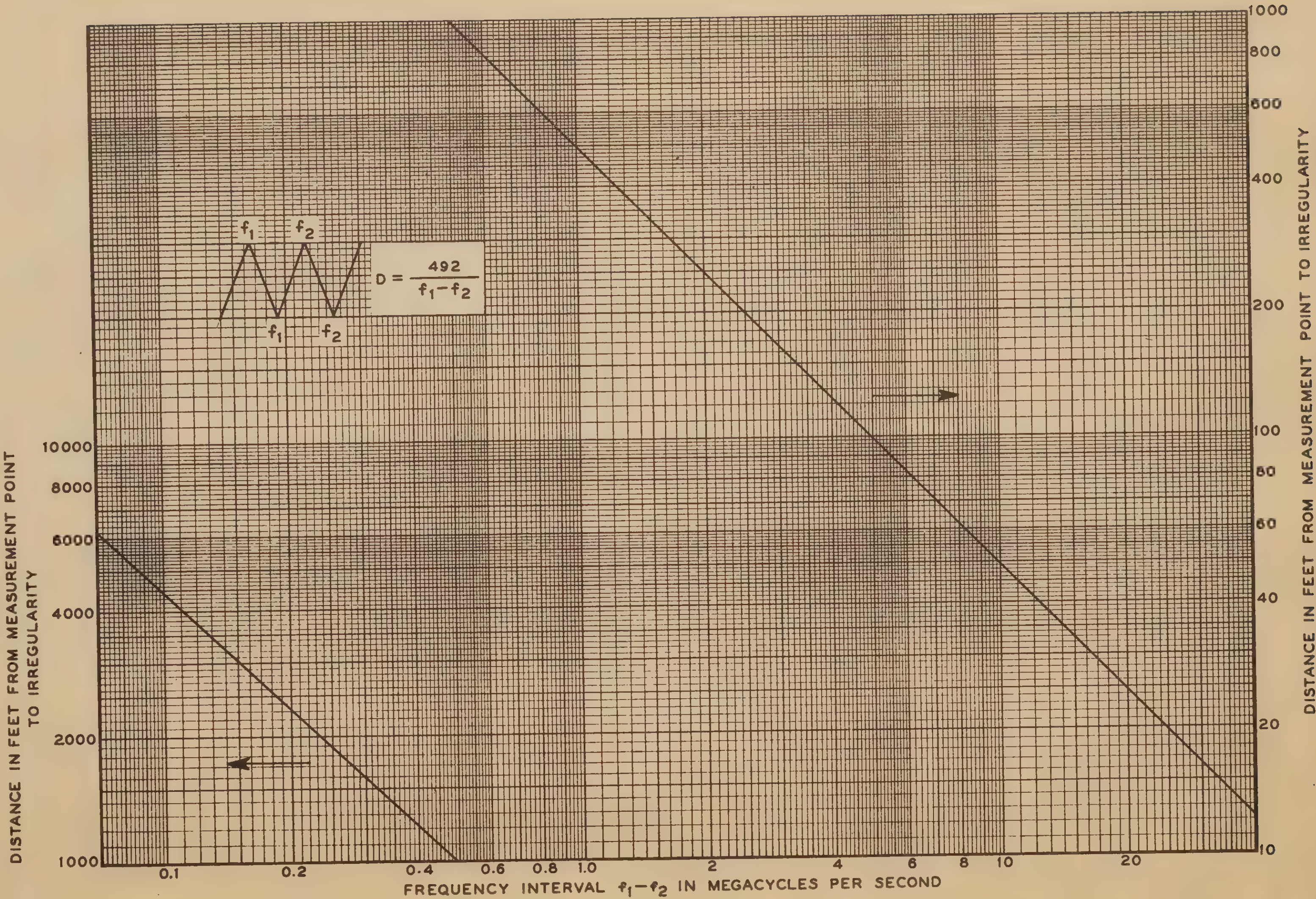


FIG. 33 — LOCATION OF IRREGULARITIES ON LIGHT VELOCITY TRANSMISSION LINES

A transmission line terminated in its characteristic impedance has a substantially uniform impedance-frequency characteristic over a wide frequency range. If the line is not terminated in its characteristic impedance, or if its constants are irregularly distributed, it will not simulate an infinite line, but reflections will be transmitted back to the sending end and combine with the initial wave to form standing waves. These standing waves will produce maxima and minima in the measured terminal impedance whose amplitudes are a function of the attenuation.

Circuit faults, impedance mismatches and line irregularities may be located from impedance-frequency data as follows. The frequency spacing $(f_2 - f_1)$ of impedance maxima or minima in cycles per second, is related to the velocity of propagation along the line v, and the distance to an impedance irregularity d, by equation (21) below, where c is the velocity of light; usually taken as 984.3×10^6 ft. per second.

$$\frac{v}{c} = \frac{2d\,(f_2 - f_1)}{c}; \quad d = \frac{v}{2(f_2 - f_1)} \tag{21}$$

The velocity ratio v/c of coaxial lines depends upon the physical design and should be determined by measurements on the line in question. In cases where a composite line is used, allowance must be made for the fact that each of the line elements has its own velocity of propagation.

Due to the fact that end effects prevent the line from being entirely open-circuited or short-circuited, the accurate determination of the velocity ratio or of the mean frequency interval, is often difficult, causing errors in the exact location of a fault. Parasitic reflections from small line irregularities such as insulators, line hardware, splices and lumped capacity effects tend to obscure the primary phenomenon in question. The method is sufficiently accurate in most cases

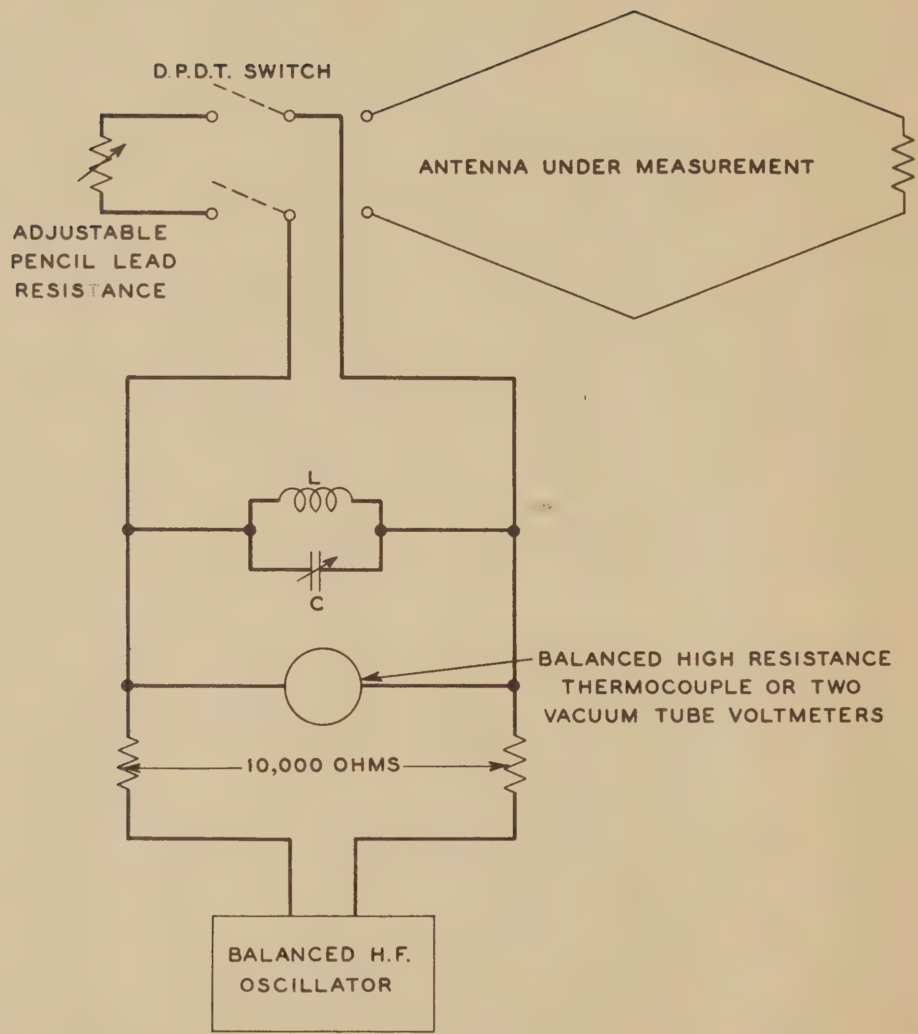


FIG. 34 — SUBSTITUTION METHOD FOR THE MEASUREMENT OF ANTENNA IMPEDANCES

to serve as an index of the source of an impedance irregularity such as distinguishing between reflections due to improper antenna termination, incorrect antenna coupling adjustment, transmission line splices, etc.

Equation (21) has been plotted on Figure 33 for the rapid location of the cause of impedance irregularities on open wire lines assuming v/c equal to unity. The curve is entered with the mean frequency spacing of impedance maxima, and the distance to the source of the irregularity from the measurement point is read on the scale of ordinates.

A method similar to that shown on Figure 34 is customarily used for measuring the impedance of antenna circuits and transmission lines[19, 7, 45]. In addition to the equipment shown on the drawing, an accurately calibrated frequency meter is required. The anti-resonant circuit LC is tuned to give a maximum voltage as measured on the sensitive thermocouple or vacuum tube voltmeter when the antenna is connected in place of R. A variable resistance, such as a pencil lead, is then substituted for the antenna at R and the capacity C is readjusted for maximum voltage. The resistor is then varied until the measured voltage is the same as it was for the antenna. The pencil lead resistance is then measured on a direct current bridge and is equal to the resistance of the antenna being measured. The reactance of the antenna may be computed from the frequency and the change in the capacity of C, with due allowance for the sign of the change.

In cases where balanced circuits are to be measured, a balance to ground must be preserved in the complete circuit of oscillator, voltage indicator, and tuned circuit. Similarly in grounded circuits a proper grounding of the measurement apparatus circuit elements prevents undesirable pickup. For the impedance ranges involved in transmission line and antenna measurements, a high output impedance oscillator is required which has sufficient power to produce an ample deflection on the voltage indicator when it is paralleled by the unknown impedance.

14A. Transmitting Antennas, Terminating Impedance

Transmitting rhombics dissipate up to half of the transmitter power in the terminating impedance and a somewhat more elaborate resistance unit is therefore required than is normally used for terminating receiving antennas (See Sec. 15A). It has been found practicable to build terminations capable of dissipating a few hundred watts by the use of assemblies of carbon rod resistors, designed to minimize the self and mutual impedances of the leads and equipment. One such arrangement of 200 watt dissipation consists of four 150 ohm carbon rod elements as shown on Figure 44.

Higher power is more effectively dissipated by means of a high attenuation transmission line. The termination end of the antenna is connected directly to an open wire balanced transmission line of the required characteristic impedance built with high resistance conductor to increase the attenuation. The electrical length is made sufficient to give an input terminal impedance substantially independent of the far end termination, which may consist of an open circuit, a resistance termination, or as is usually the case, a grounded short circuit.

It is current practice to construct these dissipative lines of magnetic material to obtain a higher linear conductor resistance at radio frequencies. No. 10 AWG stainless steel wire has been used for several recent installations, since it has corrosion resisting properties in addition to its high effective electrical resistance. Earlier lines built of ungalvanized or of thinly galvanized iron wire rusted badly, and thicker coats of galvanizing material reduced the high frequency resistance.

The actual value of the terminating impedance is generally made the value which gives the most uniform terminal impedance over the operating frequency range of the antenna. Figures 35 and 36 are good examples of the measured terminal

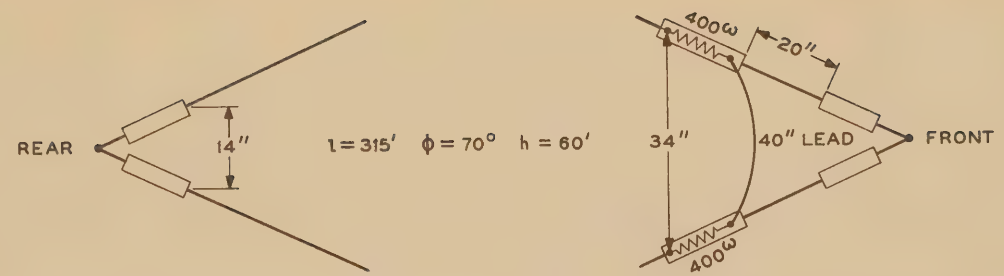
impedances of properly terminated antennas. Some typical values of terminating impedance, and the measured antenna impedance with the termination in place are shown on Table IV.

14B. <u>Transmitting Antennas, Antenna-Line Impedance Matching</u>

The problem of supplying power to a rhombic transmitting antenna is simplified by the fact that the exposure of the transmission line to radio noise sources need not be considered. Transmitting antennas are usually of the multiple wire type of construction, described in Section 15A, of about 600 ohm impedance to permit them to be matched to a conventional 600 ohm transmission line connecting the antenna to the station.

The use of open wire transmission lines is subject to a negligible loss due to transmission line radiation. It has been shown[19] that a properly balanced 2-wire line whose length is more than 20 times the wire spacing, radiates about twice the power that would be radiated by a doublet of a length equal to the wire spacing. Since the radiated power loss is independent of the line length, when longitudinals are suppressed by a proper balance to ground, it does not seriously modify the directive characteristic or power efficiency of practical rhombic antenna systems.

For comparison antennas which in general are not a part of the permanent operating plant, and may be modified for use on different frequencies, laboratory type equipment may be used. A horizontal dipole with a balanced impedance in the neighborhood of 73 ohms may be joined to a balanced-to-unbalanced network by means of a short length of twisted pair or closely spaced double conductor of the proper characteristic impedance. Where a permanent wide-frequency range is not necessary, simple impedance transformers[26] may be used, such as quarter wave lines,[19] tapped lengths,[33] line stubs[32] and other reactive networks. Where an extended frequency range must be covered, balanced impedances may be coupled by means of an exponential line[23].



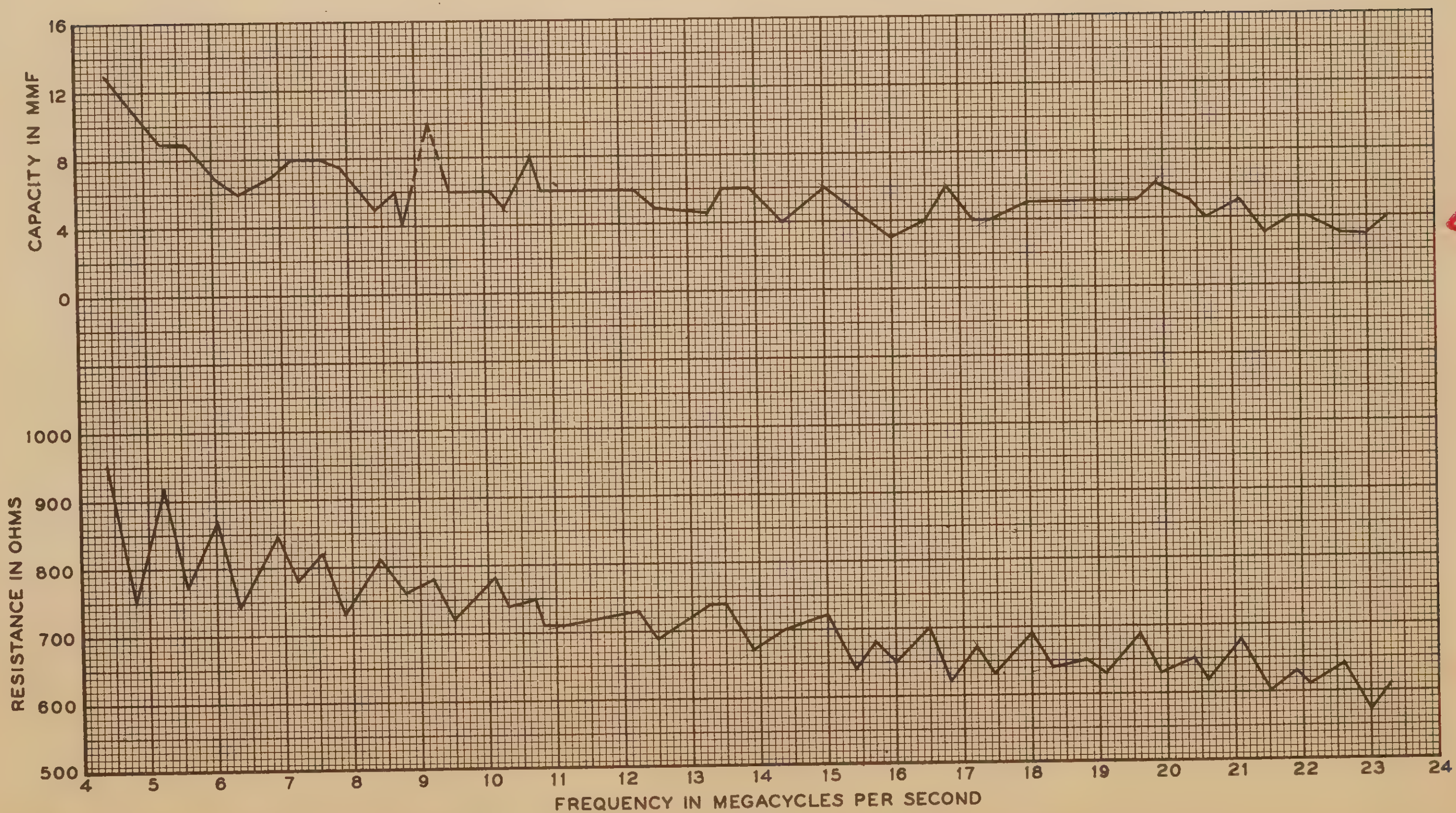


FIG. 35 — IMPEDANCE OF RHOMBIC ANTENNA

The subject of transmission line design has been discussed at length in many published papers[19,20,21,26,36] and therefore will not be included here.

15A. Receiving Antennas, Antenna and Termination Impedance

Due to the fact that a horizontal rhombic antenna does not have uniform spacing between its conducting sides, it is to be expected even in the most ideal case, that the terminal impedance will vary somewhat with frequency, regardless of the value of the terminating impedance. The magnitude of this variation and also the average impedance over the operating frequency range may be reduced by the use of multiple wire antennas mentioned in the preceding Sec. 12. The attached Figures 35 - 36 show mean impedance variations of 850 to 650 and 620 to 620 ohms respectively for single and three wire antennas terminated in impedances whose values are adjusted to give the most uniform terminal impedance. The multiple wire type of construction represents an effort to make the terminal impedance more uniform by compensating for the increased wire spacing at the side corners by the use of two or three wires which join at the ends, and are spaced as shown on Figure 43. In effect this increases the conductor diameter as the sides of the rhombus separate. A closer inspection of Figure 43 will show that the wires are actually the elements of two cones whose bases coincide. This arrangement is required to make all wire lengths identical. As mentioned in Sec. 14B, this type of low impedance antenna has also been largely used for transmitting in conjunction with open wire 600 ohm transmission lines. The rate of conductor taper, or maximum spacing of each three wire conductors at the side corners, is determined empirically by antenna terminal impedance measurements.

The variations of terminal impedance discussed above are inherent in the horizontal rhombic antenna type. Other irregularities are often caused by non-uniform or lumped capacity, due to massive fittings, insulators, etc. which lump the capacity at the corners. Improved operation is obtained by minimizing the

size of or omitting all conducting hardware, keeping the wires of the two sides of the antenna from approaching each other too closely, and maintaining a distance of several feet from the poles as shown on Figures 42 and 43.

The actual value of a transmitting antenna terminating impedance is determined as explained in Sec. 14A. Receiving antenna terminations dissipate a microscopic quantity of power in performing their intended function, and are only exposed to overload during electrical storms. Stray capacity across the resistors due to the wires approaching each other too closely, and series inductance in the connecting leads, have been found to cause serious impedance irregularities. These difficulties are reduced by dividing the total required termination resistance into several parts, as shown on Figure 42. Residual stray capacity reactance may be tuned out by a rather critical adjustment of the length of the cross connecting wire.

It should be apparent from the above that the smoothness of the antenna impedance-frequency characteristic is a function of the terminating impedance. In many practical cases it has been found necessary to use empirical design methods before the optimum termination design could be obtained. The values of the terminating resistance, frequency range, and antenna terminal impedance for several commercial radio telephone antennas are listed on the attached Table IV.

Experience gained from the analysis of the behavior of a large number of rhombic antennas of widely different dimensions, brings out the following facts. The terminal impedance-frequency characteristic of a simple single wire rhombus drops from the neighborhood of 850 to 650 ohms over a 5-20 mc frequency range, as shown on Fig. 35. The impedance and reactance in the lower part of the frequency range may be reduced by increasing the number of wires, as shown on Fig. 36.

4 to 1 on 2 octaves

~~2~~ OCTAVE
Response

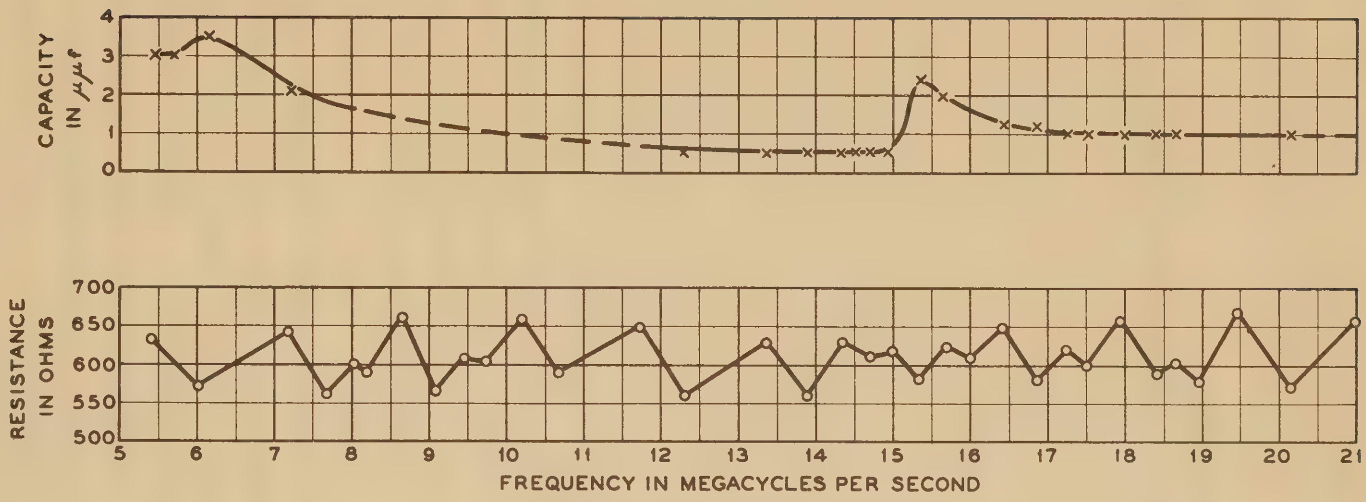


FIG. 36 — TERMINAL IMPEDANCE OF 3 WIRE RHOMBIC ANTENNA
l = 315 FT., ϕ=70, H=58 FT., 3—0.165 IN. CONDUCTORS
VERTICAL PLANE SIDE SPACING 6 FT.
HORIZONTAL PLANE END SPACING 4 FT.

Similarly at the high end of the frequency range, the impedance may be increased by increasing the spacing between the ends of the two wires at the antenna termination. Changes of from 450 to 650 ohms have been made by increasing this distance from 1 to 5 ft.

15B. Receiving Antennas

Antenna-Line Impedance Matching Coupling Networks

The transmission system joining the antenna with the receiving apparatus is similar in function to the transmission line at the transmitting station, with the additional requirement that it must be well shielded against the usual sources of radio noise. For this reason it has been Bell System practice to use a coaxial cable transmission line[19,20,21,6] extending from the radio station building to the top of the antenna coupling network pole. This cable, in earlier installations, was supported on messenger wire, in accordance with standard telephone plant practice. In more recent installations it has been found preferable to bury the cable to a depth of approximately three feet. This reduces the amplitude and frequency of temperature changes and partially eliminates the mechanical strains and abrasion resulting from expansion. In some cases it is absolutely necessary to supply a bituminous coating over the outer copper tube to avoid the effects of corrosive soils, such as salt or fresh water marsh lands.

In cases where a high insurance factor against noise induced in the transmission line is considered unnecessary, or is economically unwarranted, either a two or a four conductor open wire line may be used. On account of the high frequencies used on the line, ordinary transposition schemes are impractical. An unsuitable spacing of transpositions may increase the interference instead of reducing it. If the origin of the interference is located at a distance from the line, the reversals of phase at short line intervals makes a two wire line practically self transposed. A four wire line having the four wires

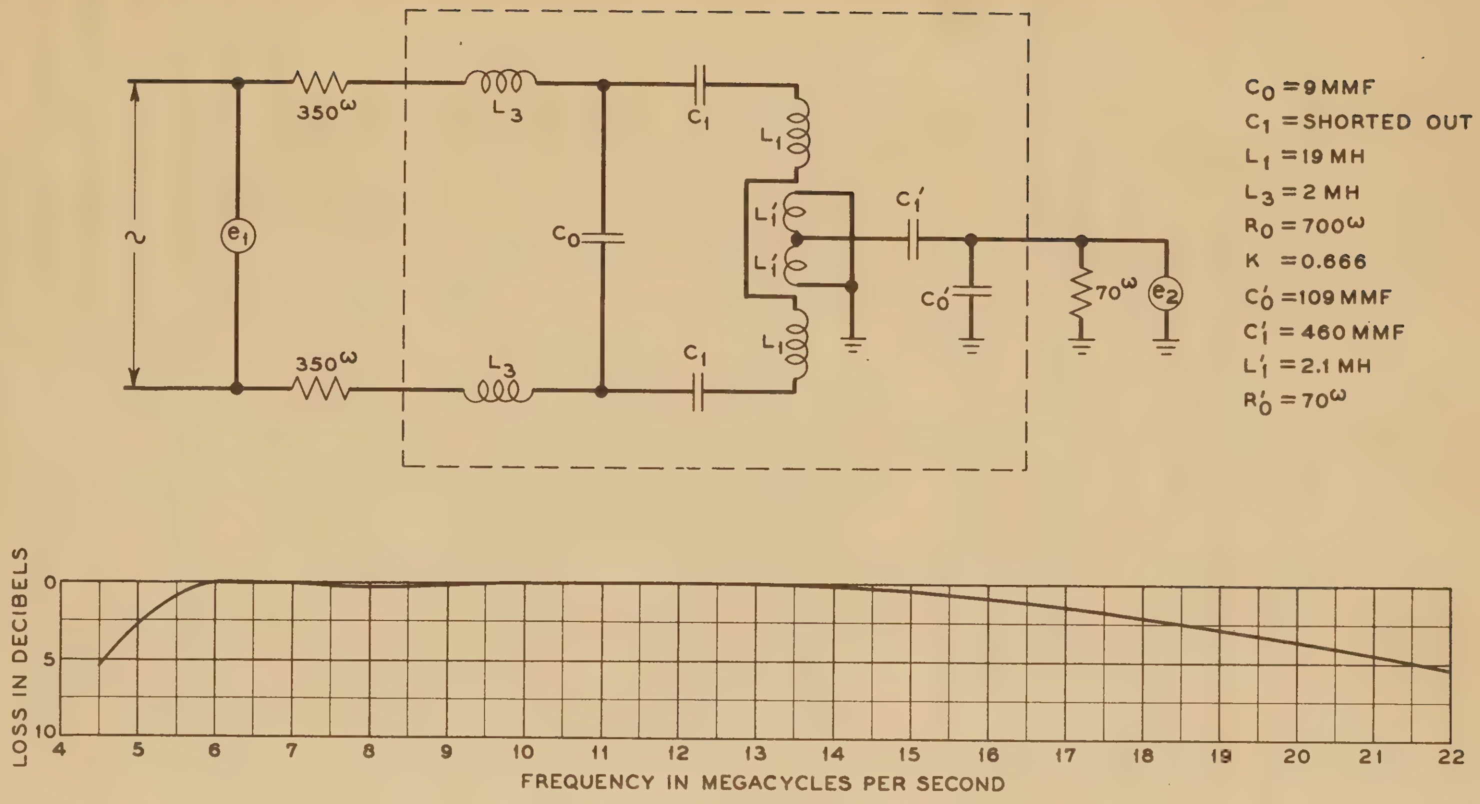


FIG. 37—FREQUENCY CHARACTERISTIC OF BROAD BAND ANTENNA COUPLING UNIT

placed at the four corners of a square and the diagonally opposite wires connected in parallel gives even better protection against induction on account of its geometrical arrangement and its low impedance.[36]

The terminal impedances to be matched also are a determining factor of the type of transmission line to be used. Practical two wire lines have an extreme characteristic impedance range of 400-800 ohms depending upon wire size and spacing, although 600 ohms has been found to be the most practical impedance[19]. Four wire lines with the conductors at the corners of a square, have practical impedances in the neighborhood of 200-300 ohms. The coaxial type of line used in radio has an impedance in the neighborhood of 72 ohms for minimum transmission loss[20].

It is apparent from the above that a 600 ohm double wire line may be used in conjunction with a multiple wire 600 ohm impedance antenna with no coupling transducer. Similarly a 300 ohm four wire line may be used to connect the radio station with two 600 ohm antennas connected in parallel.

Coaxial lines of 72 ohm impedance require a coupling transducer to avoid the reflection losses which would result from connecting them directly to antennas having terminal impedances in the neighborhood of 600-800 ohms. This transducer also serves to pass from the balanced-to-ground antenna circuit to the grounded coaxial line.

Early types of coupling networks consisted of tuned circuits coupled by mutual inductance, such as shown on Figure 37. One of the principal disadvantages of this type was that its operating frequency range did not take full advantage of the band width of a rhombic antenna. A more recent design is shown on Figure 38. This unit incorporates a metallic dust core transformer, inductances to tune out the residual antenna capacity reactance, lightning protectors, and circuits to permit direct current maintenance testing of antenna continuity. The transformer has been designed to have a high-side terminal impedance

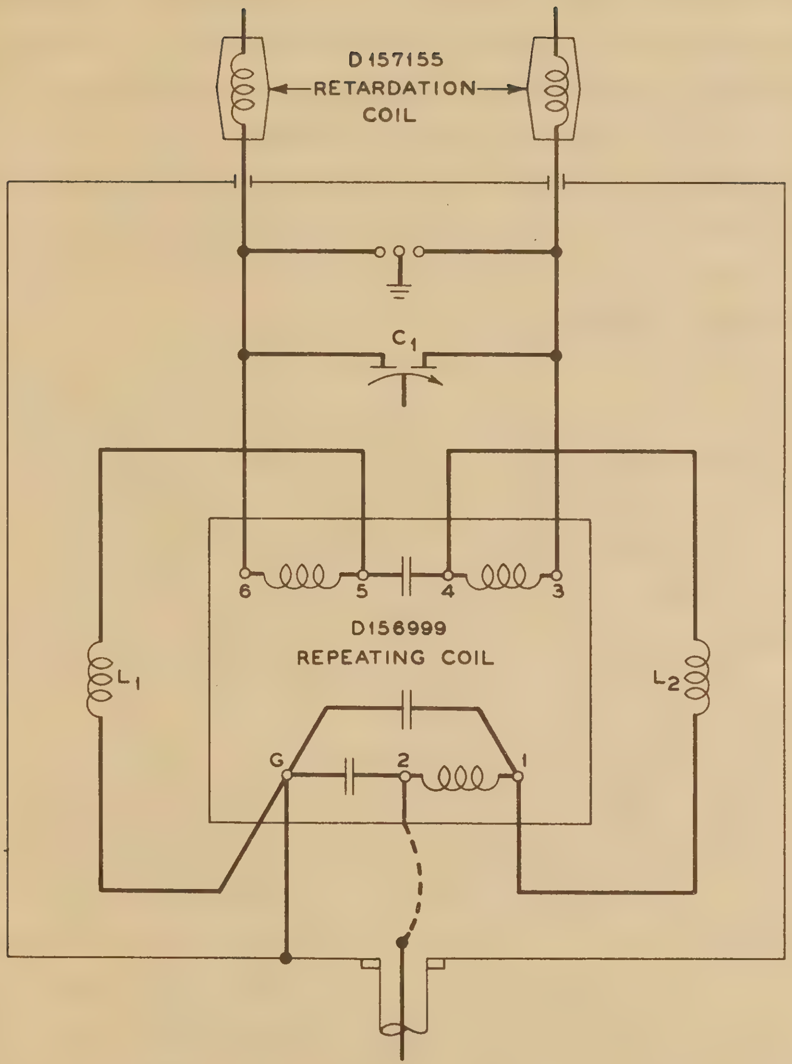


FIG.38— D157000 ANTENNA COUPLING TRANSFORMER
WIRING DIAGRAM

which coincides with the sloping impedance characteristic of a single wire rhombic antenna. The transmission loss of this network is less than 0.5 db over a 4-22 megacycle frequency range.

A general analysis of the advantages of coaxial vs. open wire lines for receiving, indicates that coaxial lines give the ultimate protection against noise induction from sources within or adjacent to the station building, but require an antenna coupling transducer between the line and antenna. Open wire lines are less effectively shielded against interference but are cheaper and require no coupling transducer.

16. Maintenance Provisions

In maintaining an extended antenna system it is desirable to have a method for checking the circuit continuity from the station building. This facility may be provided on both transmitting and receiving antennas by arranging the circuits to give a continuous direct current path around the loop of transmission line, coupling unit, antenna, termination and return. Comparison of the resistance measured in these circuits with data taken immediately after construction is often of assistance in locating faults.

It is current design practice to use No. 6 A.W.G. (162 mil.) 40% Copperweld wire with a 2433 lb. breaking strength, in combination with insulators which are slightly weaker, such as 1900 lb. Isolantite strain insulators. At stations in the vicinity of New York, which is in the so-called heavy loading district, sags, tensions and spans are designed to withstand twice the load which would result from the vertical weight of the conductor plus the added weight of a layer of ice 1/2" in radial thickness, combined with a horizontal wind pressure of 8 lbs. per sq. ft. on the projected area of the ice covered conductor, at a minimum temperature of 0° F[26,40]. This design loading has been considered a reasonable compromise at the New Jersey radio stations, and no provision has been made for sleet melting on the antenna proper.

The collection of ice and sleet on open wire transmission lines to the transmitting antennas is not a serious mechanical hazard but modifies the line propagation constant and characteristic impedance to such an extent that its removal is desirable. For this purpose some of the transmitting antennas have been supplied with sleet melting equipment consisting of a variable voltage 200 volt 150 amp. d-c motor generator and a switch for short circuiting the antenna termination. The power dissipated as heat is divided between the antenna and transmission line by using a 165-mil copper transmission line to feed a 162 mil 40% conductivity Copperweld three wire antenna. Care must be taken to avoid damage to the wire materials by the application of too much heat.

17. Lightning Protection

Damage from electrical storms may be minimized by the usual expedients employed on exposed communication and transmission lines. At receiving locations, where a coaxial line connects the antenna to the apparatus, spark gaps or carbon protector blocks are installed between each side of the antenna output and ground as shown on Figure 38. On all except direct hits this protection has been found sufficient to prevent appreciable damage to the coupling unit or station apparatus. If an open-wire line is to be used instead of the coaxial, an additional set of protectors will be required where the line enters the station building.

Transmitting stations are protected by horn gaps to ground across the termination and across the open-wire line at the point where it enters the building as shown on Fig. 43. Additional protection is furnished by grounding the short-circuited far end of the dissipative terminating line.

Pole damage has been reduced by the use of sectionalized grounded conductors on the poles. The top of these conductors may be terminated in an elevated rod, or it may

Good to use

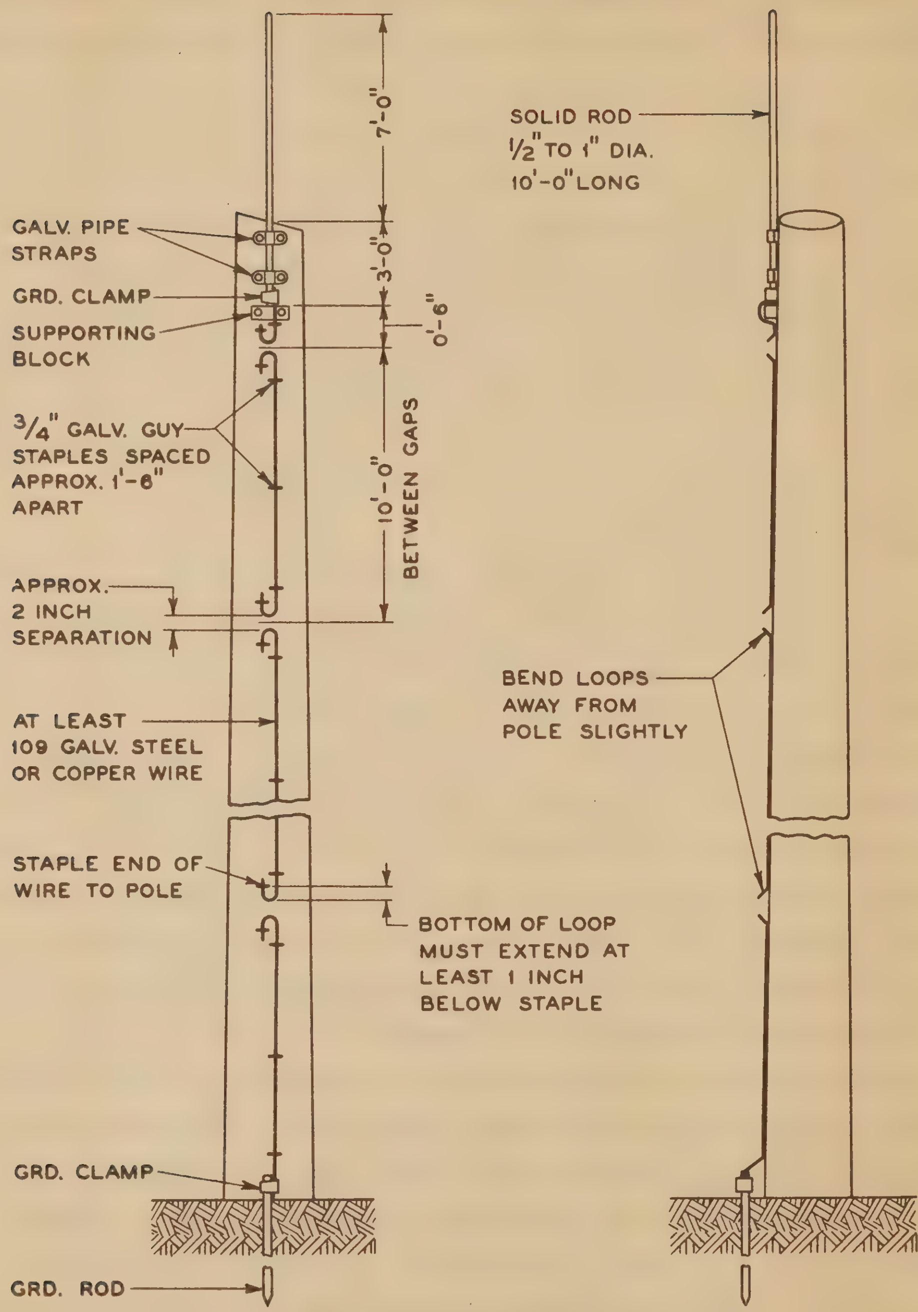


FIG.39 — LIGHTNING ROD ARRANGEMENT FOR RHOMBIC ANTENNA POLES

be looped over the pole top. Available data indicate no difference in the protection afforded by these methods. The grounded conductor is divided into ten foot sections by two inch gaps as shown on Figure 39.

Due to the necessity of preventing the lightning protective apparatus from interfering with the transmission properties of the system, special precautions must be taken to avoid the introduction of lumped capacity irregularities, such as massive horn gaps or high capacity receiving antenna protectors. By minimizing the physical size of these elements, installing them in the most desirable circuit location, and giving attention to the length and arrangement of the connecting leads, it has been found possible to reduce any tendency to cause impedance irregularities.

On account of the impossibility of securing complete protection with any practical means, it has been found expedient to maintain a stock of especially vulnerable parts such as insulators, wire, coupling transformers and protectors.

18. Construction

The construction details of two types of recently constructed horizontal rhombic antennas are shown on the attached Figures 42-43. The mechanical requirements are similar to those commonly applied to the design of overhead wire lines and will be reviewed very briefly[26,40].

In general it has been found desirable to limit the maximum working tension to 50% of the ultimate strength of the material. The wire tension in pounds may be computed in terms of the span length S and the deflection d, expressed in feet; and the loading w in pounds per foot, by means of equation (22).

$$t = \frac{S^2 w}{8d} \qquad (22)$$

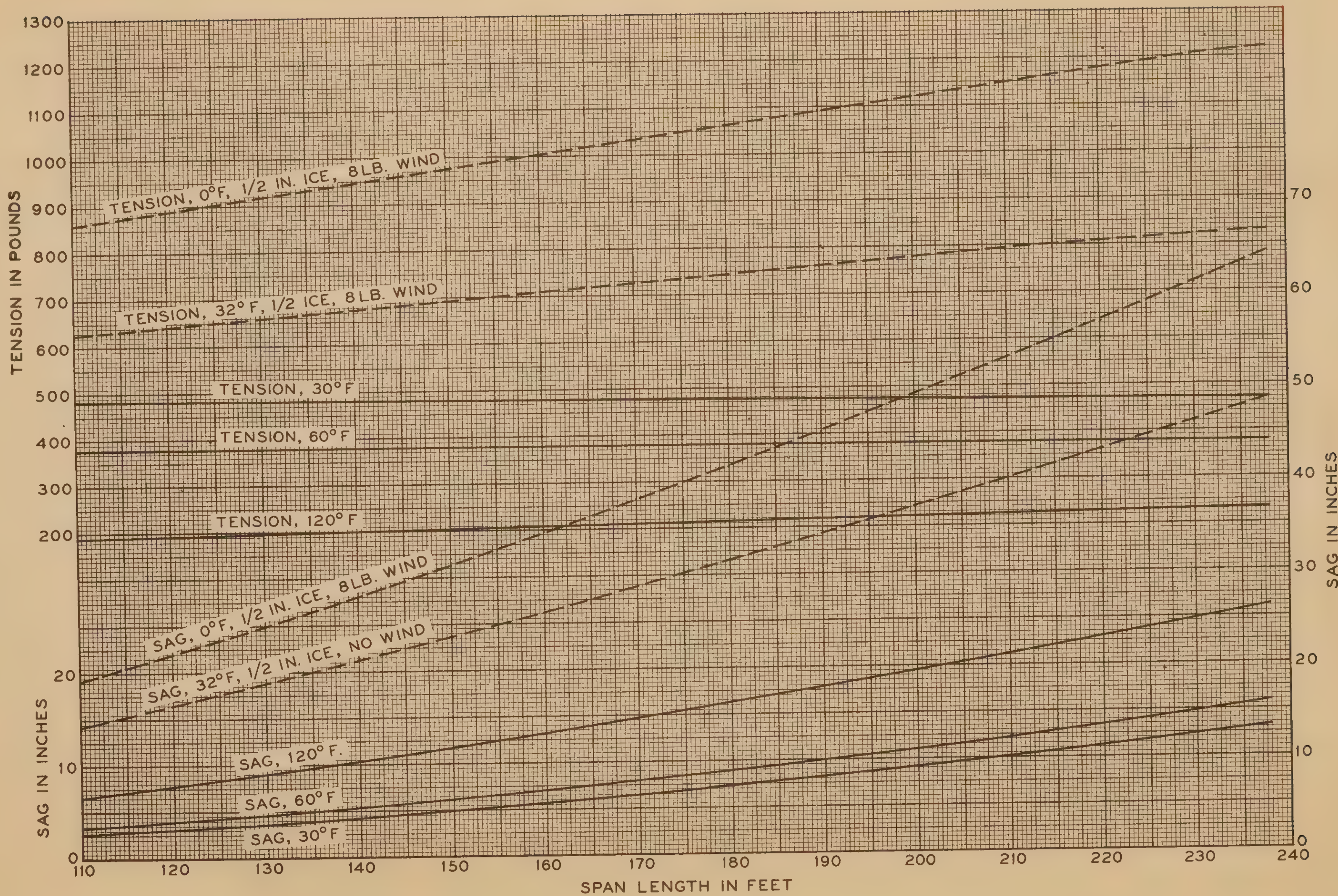


FIG.40 — FINAL SAGS AND TENSIONS FOR NO.6 A.W.G., HIGH STRENGTH, 40% COND., COPPERWELD SOLID BARE WIRE.
BREAKING STRENGTH 2433 LBS.

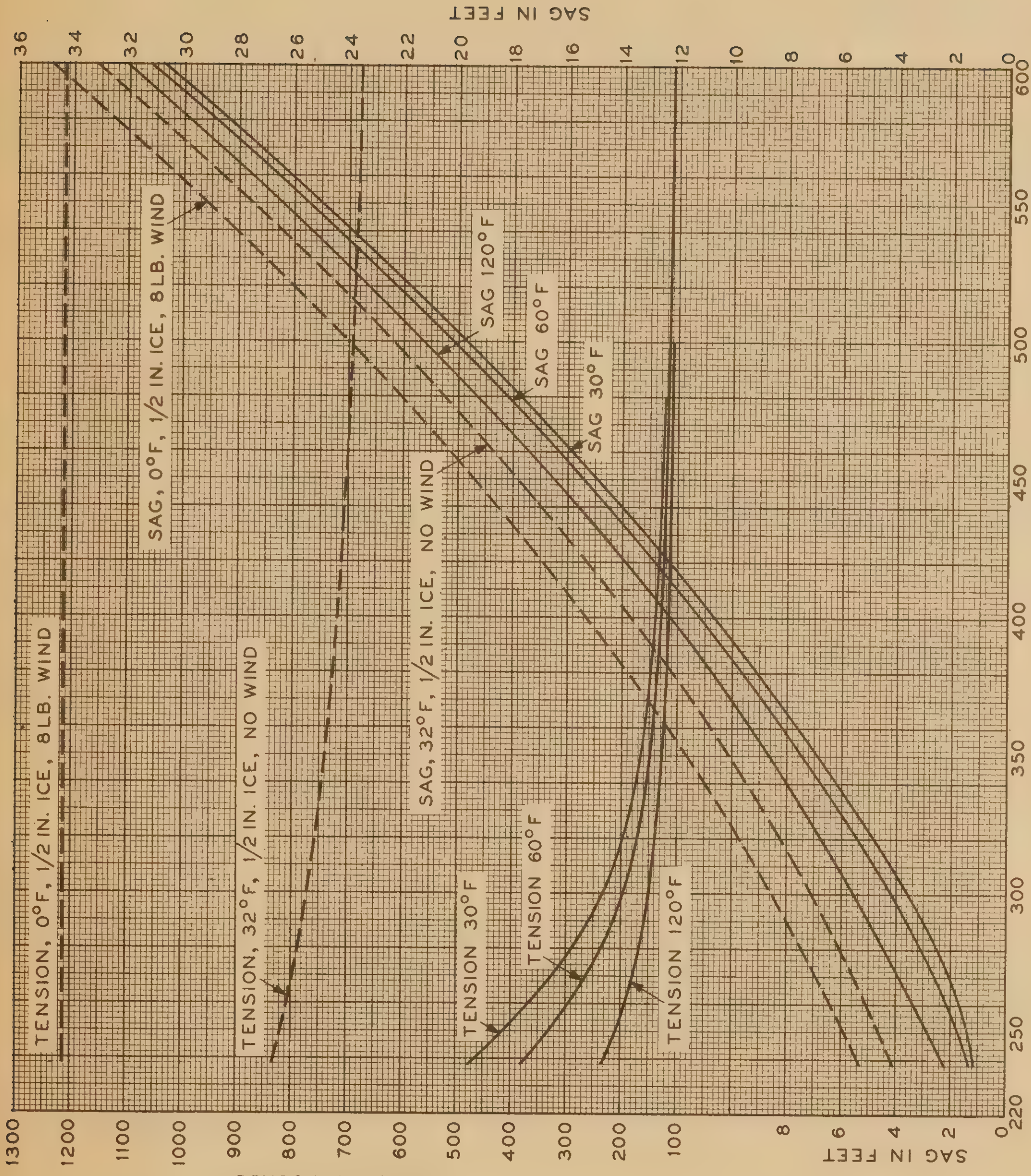


FIG. 41 — FINAL SAGS AND TENSIONS FOR NO.6 A. W. G.. HIGH STRENGTH 40% COND.. COPPERWELD SOLID BARE WIRE.
BREAKING STRENGTH 2433 LBS.

Point to point transoceanic radio telephone antennas built in New Jersey have been designed to withstand heavy loading as above specified. In more southerly localities lighter loading designs are being replaced by the heavy type of construction to resist the effects of tropical storms. In cases where the wire tension may be reduced in winter and increased in summer by means of turnbuckles, it is possible to use smaller sags in the season when no sleet is expected.

Copperweld line design data plotted on Figures 40-41 show the allowable sag and tension for the required grade of construction in the heavy loading district. These data give the final sag to be used after initially loading the line to the maximum allowable tension. Nicked, kinked or temperature weakened wire must of course be avoided. In localities where heavy loading construction is used to withstand wind storms and sleet is rare, tensions considerably higher than those shown on Figures 40-41 are sometimes used with reasonable safety.

Strain insulators of 1800 lb. breaking strength are used to support single wire spans, and 6000 lb. breaking strength insulators carry the resultant load of three wire spans. These insulator strengths fall midway between the assumed wire safe loading and the maximum allowable wire strength of 2433 lbs.

The general subject of line and antenna construction includes such matters as pole setting, guying, pole attachment and open wire construction. These items are discussed in readily available handbooks,[26,40] and in books on overhead wire line construction[25].

TABLE I

NOMENCLATURE

l = Length of one side of rhombus, meters.

φ = One half of the interior side angle of the rhombus.

ξ_2) used interchangeably to designate the
) vertical angle between the ground plane and an
Δ) incident wave, degrees.

f = frequency in cycles.

β = horizontal angle between a direction under consideration and the longer diagonal of the rhombus, degrees.

H = height of rhombus above the ground, meters.

E = R.M.S. Resultant field strength, microvolts per meter.

E_o = R.M.S. Directly transmitted field strength.

K = amplitude change during reflection from the ground (for perfectly conducting earth with horizontal polarization, $K = 1$)

ϵ = dielectric constant of the ground, E.S.U.

V)
H) Subscripts referring to vertical or horizontal polarization.

ψ = phase advance at reflection $\pm \pi$ radians.

λ = wave length, meters.

σ = conductivity of the ground, E.S.U.

P = Received power, micromicrowatts.

I_s = Transmitting antenna current, amperes.

R_a = Radiation resistance, ohms.

Z_o = Characteristic impedance, or antenna terminal impedance, ohms.

R = Load resistance, ohms.

γ = Phase difference between direct and reflected wave $\pm \pi$ radians.

K_F = Radiation function.

Φ = Radiation intensity, watts per sq. meter at one meter.

D = Directivity factor.

$K_1 = 1 - \cos\Delta \sin(\varphi + \beta)$

$K_2 = 1 - \cos\Delta \sin(\varphi - \beta)$

A and B = Dimensional constants

TABLE II

TYPICAL GROUND CONSTANTS

Type or Location of Ground	Dielectric Constant	Conductivity E.S.U.	Authority
1. Sea Water	80	$.9 \times 10^{10}$	Madrid Conference
2. Fresh Water	80	$.9 \times 10^{7}$	" "
3. Wet Ground	10	4.5×10^{7}	" "
4. Dry Ground	4	$.9 \times 10^{6}$	" "
5. Mid West U.S.		$.9 \times 10^{8}$	" "
6. Texas		2.7×10^{8}	" "
7. South West U.S.		3.6×10^{7}	" "
8. New England		1.8×10^{7}	" "
9. Ocean near N. J.		3.9×10^{10}	L. A. Wooten
10. N. J. Lakes		6×10^{7}	" " "
11. Lake Michigan		2.2×10^{8}	" " "
12. Holmdel, N.J.	15-25	$1\text{-}2 \times 10^{8}$	C.B. Feldman
13. Netcong, N.J.	5-10	$1\text{-}3 \times 10^{7}$	" " "
14. Japan	15	1.8×10^{7}	" " "
15. Philippines	12	2.7×10^{7}	" " "
16. Dry Soil	3-4	1×10^{5}	Smith - Rose
17. Moist Soil	30-40	$1\text{-}2 \times 10^{8}$	" "

TABLE III

MEASURED SIGNAL GAINS
EXPERIMENTAL AND OPERATING RHOMBIC ANTENNAS

Location	Measured With	Dimensions			f	Gain	Referred to
		ℓ	h	φ	mc	db	
		Transmitting Antennas					
Dixon Calif.	Manila, P.I.	269'	80'	66°	7.61	12	($\lambda/2$ Horizontal
					10.84	15.8	(Dipole at (height of (antenna
Lawrenceville, N. J.	England	512'	95'	75°	9.4	8.0	($\lambda/2$ Vertical (with midpoint
					13.4	9.0	(1.25λ above (ground
Lawrenceville, N. J.	Bermuda	278'	80'	64°	6.8	8.0	"
					10.7	13.0	"
		Receiving Antennas					
Holmdel, N. J. (Long time average)	England	315'	58'	70°	16.25	11	($\lambda/2$ Horizontal (Dipole
						19	($\lambda/2$ Vertical (at Ground
Netcong, N. J.	England	572'	60'	(71-77) (Variable)	9.0	10.7	($\lambda/2$ Vertical (at Ground
					14.4	12.5	"

Note: The above data are representative of the average signal gains observed during normally existing combinations of fixed antenna directivity with variable transmission path angles and polarization conditions. They show the relationship between each rhombic and its individual reference antenna, but are not mutually comparable.

TABLE IV

TERMINAL AND TERMINATION IMPEDANCE MEASURED ON TYPICAL SINGLE WIRE RHOMBIC ANTENNAS.

Antennas at Netcong, N.J.

Antenna	Dimension	Frequency Range m.c.	Antenna Termination ohms	Antenna Impedance for stated Termination Max.	ohms	Min.
Lima 3700 Miles	ℓ=300 ft. φ=65°, h=54'	10-21	850	600	515	440
B.A. 5300 Miles	ℓ=300 ft. φ=65°, h=55'	10.1-21.2	803	620	541	470
Rio 4900 Miles	ℓ=300 ft. φ=65°, h=57'	10.0-21.2	867	600	537	480
European No. 1 3500 Miles	ℓ=570 ft. φ=75°, h=54'	4.9-20.0	680	760	592	460
European No. 2	ℓ=570 ft. φ=75°, h=58'	4.1-19.9	710	790	622	490
European No. 3	ℓ=570 ft. φ=75°, h=58'	4.0-20.0	708	890	648	470
Italian 4300 Miles	ℓ=315 ft. φ=70°, h=60'	10-20	800	-	-	-

Antennas at Hialeah, Fla.

Antenna	Dimension	Frequency Range m.c.	Antenna Termination ohms	Antenna Impedance for stated Termination Max.	ohms	Min.
Columbia 1100 Miles 1300 " 1500 "	ℓ=143.8 ft. φ=55°, h=62	10.2-14.5	900	680	606	540
Venezuela 1500 Miles	ℓ=155.5 ft. φ=55°, h=64	9.05-12.55	900	605	546	490
Central American 1000 Miles	ℓ=143.9 ft. φ=55°, h=64	9.25-13.10	900	720	680	640
Panama 1200 Miles	ℓ=143.9 ft. φ=55°, h=64	12.55-16.5	900	740	680	620
Santo Domingo 800 Miles	ℓ=131 ft. φ=58°, h=62'	10-15	950	-	-	-

TABLE V

LOADING IN POUNDS PER FOOT.
SOLID COPPER OR COPPERWELD WIRE.

AWG Number	Load Due to Wire Only	Resultant Light Loading	Resultant Medium Loading	Weight of Conductor plus 1/2 inch ice	Resultant Heavy Loading
#6	.08	.18	.49	.49	.92
8	.05	.14	.45	.44	.87

LOADING CLASSIFICATIONS

Loading District	Maximum Assumed Loading
Heavy	1/2 inch ice, 8 lb. wind at 0°F.
Medium	1/4 " ", 8 lb. " " 15°F.
Light	No ice, 12 lb. wind at 30°F.

TABLE VI.

$\frac{\sin x}{x}$ in Terms of x Radians

x	$\frac{\sin x}{x}$	x	$\frac{\sin x}{x}$	x	$\frac{\sin x}{x}$	x	$\frac{\sin x}{x}$	x	$\frac{\sin x}{x}$
0	1	0,50	0,95885 108	1,00	0,84147 098	1,50	0,66499 666	2,00	0,45464 871
0,01	0,99998 333	1	721 029	1	3844 737	1	6102 812	1	5029 381
2	93 333	2	553 873	2	3540 002	2	5704 615	2	4593 722
3	85 001	3	383 649	3	3232 912	3	5305 094	3	4157 918
4	73 335	4	210 369	4	2923 483	4	4904 275	4	3721 993
5	58 339	5	034 042	5	2611 736	5	4502 178	5	3285 969
6	40 011	6	0,94854 678	6	2297 687	6	4098 828	6	2849 871
7	18 353	7	672 289	7	1981 356	7	3694 247	7	2413 723
8	0,99893 367	8	486 886	8	1662 760	8	3288 458	8	1977 547
9	65 055	9	298 478	9	1341 919	9	2881 485	9	1541 368
0,10	0,99833 417	0,60	0,94107 079	1,10	0,81018 851	1,60	0,62473 350	2,10	0,41105 208
1	798 455	1	0,93912 698	1	0693 575	1	2064 077	1	0669 091
2	760 173	2	715 348	2	0366 111	2	1653 688	2	0233 042
3	718 571	3	515 041	3	0036 477	3	1242 207	3	0,39797 082
4	673 653	4	311 788	4	0,79704 693	4	0829 657	4	9361 235
5	625 422	5	105 601	5	9370 777	5	0416 062	5	8925 525
6	573 879	6	0,92896 493	6	9034 751	6	0001 445	6	8489 976
7	519 029	7	684 476	7	8696 632	7	0,59585 829	7	8054 608
8	460 874	8	469 562	8	8356 442	8	9169 238	8	7619 447
9	399 418	9	251 766	9	8014 199	9	8751 695	9	7184 516
0,20	0,99334 665	0,70	0,92031 098	1,20	0,77669 924	1,70	0,58333 224	2,20	0,36749 837
1	266 619	1	0,91807 573	1	7323 636	1	7913 848	1	6315 433
2	195 283	2	581 204	2	6975 357	2	7493 591	2	5881 328
3	120 662	3	352 005	3	6625 106	3	7072 476	3	5447 544
4	042 761	4	119 988	4	6272 903	4	6650 528	4	5014 104
5	0,98961 584	5	0,90885 168	5	5918 770	5	6227 768	5	4581 031
6	877 135	6	647 559	6	5562 726	6	5804 223	6	4148 348
7	789 421	7	407 174	7	5204 792	7	5379 914	7	3716 077
8	698 446	8	164 028	8	4844 989	8	4954 866	8	3284 242
9	604 216	9	0,89918 136	9	4483 338	9	4529 102	9	2852 864
0,30	0,98506 736	0,80	0,89669 511	1,30	0,74119 860	1,80	0,54102 646	2,30	0,32421 966
1	406 012	1	418 170	1	3754 576	1	3675 522	1	1991 570
2	302 050	2	164 126	2	3387 508	2	3247 754	2	1561 700
3	194 857	3	0,88907 394	3	3018 675	3	2819 366	3	1132 377
4	084 439	4	647 990	4	2648 100	4	2390 380	4	0703 624
5	0,97970 802	5	385 930	5	2275 804	5	1960 822	5	0275 462
6	853 954	6	121 228	6	1901 809	6	1530 714	6	0,29847 914
7	733 901	7	0,87853 901	7	1526 136	7	1100 081	7	9421 002
8	610 650	8	583 964	8	1148 807	8	0668 946	8	8994 747
9	484 209	9	311 432	9	0769 843	9	0237 334	9	8569 172
0,40	0,97354 586	0,90	0,87036 323	1,40	0,70389 266	1,90	0,49805 268	2,40	0,28144 299
1	221 787	1	0,86758 653	1	0007 099	1	9372 771	1	7720 149
2	085 822	2	478 437	2	0,69623 364	2	8939 868	2	7296 744
3	0,96946 698	3	195 693	3	9238 081	3	8506 583	3	6874 105
4	804 424	4	0,85910 436	4	8851 274	4	8072 939	4	6452 254
5	659 008	5	622 685	5	8462 965	5	7638 960	5	6031 212
6	510 458	6	332 455	6	8073 176	6	7204 669	6	5611 001
7	358 784	7	039 764	7	7681 929	7	6770 092	7	5191 642
8	203 995	8	0,84744 630	8	7289 246	8	6335 250	8	4773 156
9	046 100	9	447 069	9	6895 151	9	5900 169	9	4355 563

TABLE VI. (CONTINUED)

$\frac{\sin x}{x}$ in Terms of x Radians

x	$\frac{\sin x}{x}$	x	$\frac{\sin x}{x}$	x	$\frac{\sin x}{x}$	x	$\frac{\sin x}{x}$	x	$\frac{\sin x}{x}$
2,50	0,23938 886	3,00	0,04704 000	3,50	0,10022 378	4,00	0,18920 062	4,50	0,21722 892
1	3523 144	1	4359 242	1	0260 117	1	0,19034 937	1	20 381
2	3108 359	2	4016 333	2	0495 482	2	147 342	2	15 714
3	2694 551	3	3675 287	3	0728 467	3	257 279	3	08 900
4	2281 741	4	3336 118	4	0959 065	4	364 750	4	0,21699 951
5	1869 950	5	2998 841	5	1187 272	5	469 759	5	88 876
6	1459 197	6	2663 469	6	1413 082	6	572 309	6	75 685
7	1049 503	7	2330 017	7	1636 488	7	672 403	7	60 390
8	0640 888	8	1998 497	8	1857 485	8	770 043	8	42 999
9	0233 372	9	1668 925	9	2076 069	9	865 234	9	23 525
2,60	0,19826 976	3,10	0,01341 312	3,60	0,12292 235	4,10	957 978	4,60	0,21601 978
1	9421 718	1	1015 672	1	2505 977	1	0,20048 281	1	578 369
2	9017 620	2	0692 018	2	2717 290	2	136 145	2	552 709
3	8614 700	3	0370 364	3	2926 172	3	221 575	3	525 010
4	8212 978	4	0050 721	4	3132 617	4	304 575	4	495 282
5	7812 473	5	0,00266 897	5	3336 621	5	385 149	5	463 536
6	7413 205	6	0582 478	6	3538 180	6	463 303	6	429 786
7	7015 194	7	0896 010	7	3737 290	7	539 040	7	394 041
8	6618 457	8	1207 481	8	3933 949	8	612 366	8	356 315
9	6223 014	9	1516 879	9	4128 152	9	683 286	9	316 618
2,70	5828 884	3,20	0,01824 192	3,70	0,14319 896	4,20	751 804	4,70	0,21274 963
1	5436 086	1	2129 408	1	509 178	1	817 927	1	231 362
2	5044 639	2	2432 517	2	695 996	2	881 659	2	185 827
3	4654 560	3	2733 506	3	880 346	3	943 007	3	138 371
4	4265 868	4	3032 364	4	0,15062 227	4	0,21001 977	4	089 005
5	3878 582	5	3329 081	5	241 635	5	058 573	5	037 743
6	3492 719	6	3623 646	6	418 569	6	112 803	6	0,20984 597
7	3108 298	7	3916 048	7	593 027	7	164 673	7	929 580
8	2725 336	8	4206 276	8	765 006	8	214 188	8	872 704
9	2343 852	9	4494 321	9	934 505	9	261 357	9	813 983
2,80	1963 863	3,30	0,04780 173	3,80	0,16101 523	4,30	306 185	4,80	0,20753 429
1	1585 386	1	5063 820	1	266 059	1	348 679	1	691 057
2	1208 438	2	5345 254	2	428 111	2	388 846	2	626 878
3	0833 038	3	5624 464	3	587 677	3	426 694	3	560 907
4	0459 203	4	5901 442	4	744 759	4	462 230	4	493 156
5	0086 948	5	6176 178	5	899 354	5	495 462	5	423 640
6	0,09716 291	6	6448 665	6	0,17051 463	6	526 396	6	352 372
7	9347 249	7	6718 888	7	201 085	7	555 041	7	279 365
8	8979 839	8	6986 844	8	348 220	8	581 404	8	204 634
9	8614 076	9	7252 522	9	492 868	9	605 495	9	128 192
2,90	8249 977	3,40	0,07515 915	3,90	0,17635 030	4,40	627 320	4,90	0,20050 053
1	7887 558	1	7777 013	1	774 705	1	646 888	1	0,19970 232
2	7526 836	2	8035 809	2	911 895	2	664 208	2	888 742
3	7167 826	3	8292 295	3	0,18046 600	3	679 289	3	805 599
4	6810 544	4	8546 463	4	178 822	4	692 138	4	720 815
5	6455 005	5	8798 305	5	308 560	5	702 765	5	634 405
6	6101 225	6	9047 814	6	435 817	6	711 180	6	546 385
7	5749 220	7	9294 983	7	560 594	7	717 390	7	456 768
8	5399 004	8	9539 804	8	682 893	8	721 406	8	365 570
9	5050 592	9	9782 272	9	802 715	9	723 236	9	272 804

TABLE VI. (CONTINUED)

$\frac{\sin x}{x}$ in Terms of x Radians

x	$\frac{\sin x}{x}$	x	$\frac{\sin x}{x}$	x	$\frac{\sin x}{x}$	x	$\frac{\sin x}{x}$	x	$\frac{\sin x}{x}$
5,00	0,19178 485	5,50	0,12828 006	6,00	0,04656 925	6,50	0,03309 538	7,00	0,09385 523
1	082 630	1	675 471	1	489 184	1	454 300	1	479 210
2	0,18985 252	2	522 224	2	321 553	2	598 273	2	571 684
3	886 366	3	368 281	3	154 046	3	741 446	3	662 940
4	785 988	4	213 659	4	0,03986 679	4	883 807	4	752 971
5	684 132	5	058 376	5	819 468	5	0,04025 346	5	841 772
6	580 814	6	0,11902 447	6	652 426	6	166 052	6	929 340
7	476 050	7	745 890	7	485 571	7	305 913	7	0,10015 668
8	369 854	8	588 722	8	318 917	8	444 919	8	100 752
9	262 242	9	430 959	9	152 478	9	583 059	9	184 587
5,10	0,18153 229	5,60	272 619	6,10	0,02986 271	6,60	0,04720 324	7,10	267 170
1	042 831	1	113 717	1	820 309	1	856 701	1	348 494
2	0,17931 064	2	0,10954 272	2	654 608	2	992 182	2	428 557
3	817 943	3	794 300	3	489 183	3	0,05126 755	3	507 354
4	703 483	4	633 818	4	324 048	4	260 412	4	584 881
5	587 702	5	472 842	5	159 218	5	393 141	5	661 134
6	470 613	6	311 390	6	0,01994 708	6	524 933	6	736 110
7	352 234	7	149 478	7	830 532	7	655 778	7	809 804
8	232 581	8	0,09987 122	8	666 704	8	785 666	8	882 213
9	111 668	9	824 341	9	503 239	9	914 589	9	953 335
5,20	0,16989 513	5,70	661 150	6,20	0,01340 152	6,70	0,06042 536	7,20	0,11023 165
1	866 130	1	497 566	1	177 456	1	169 499	1	091 700
2	741 538	2	333 606	2	015 165	2	295 467	2	158 938
3	615 750	3	169 287	3	0,00853 294	3	420 433	3	224 876
4	488 785	4	004 624	4	691 857	4	544 387	4	289 511
5	360 657	5	0,08839 636	5	530 867	5	667 320	5	352 840
6	231 384	6	674 338	6	370 339	6	787 223	6	414 861
7	100 981	7	508 747	7	210 286	7	910 089	7	475 571
8	0,15969 465	8	342 380	8	050 721	8	0,07029 908	8	534 968
9	836 852	9	176 753	9	0,00108 341	9	148 672	9	593 051
5,30	0,15703 159	5,80	010 382	6,30	0,00266 887	6,80	0,07266 373	7,30	649 817
1	568 403	1	0,07843 785	1	424 905	1	383 003	1	705 264
2	432 599	2	676 977	2	582 379	2	498 553	2	759 391
3	295 764	3	509 975	3	739 298	3	613 017	3	812 195
4	157 915	4	342 795	4	895 649	4	726 385	4	863 676
5	019 068	5	175 453	5	0,01051 417	5	838 651	5	913 833
6	0,14879 241	6	007 967	6	206 591	6	949 807	6	962 663
7	738 449	7	0,06840 351	7	361 157	7	0,08059 846	7	0,12010 166
8	596 709	8	672 623	8	515 102	8	168 760	8	056 341
9	454 038	9	504 798	9	668 415	9	276 542	9	101 188
5,40	0,14310 453	5,90	336 893	6,40	0,01821 081	6,90	0,08383 185	7,40	144 704
1	165 971	1	168 923	1	973 090	1	488 682	1	186 890
2	020 608	2	000 905	2	0,02124 428	2	593 027	2	227 746
3	0,13874 380	3	0,05832 854	3	275 083	3	696 213	3	267 270
4	727 305	4	664 787	4	425 043	4	798 233	4	305 463
5	579 400	5	496 720	5	574 296	5	899 081	5	342 325
6	430 681	6	328 667	6	722 830	6	998 750	6	377 855
7	281 166	7	160 646	7	870 633	7	0,09097 235	7	412 054
8	130 870	8	0,04992 671	8	0,03017 693	8	194 530	8	444 922
9	0,12979 811	9	824 759	9	163 999	9	290 627	9	476 459

TABLE VI. (CONTINUED)

$\frac{\sin x}{x}$ in Terms of x Radians

x	$\frac{\sin x}{x}$	x	$\frac{\sin x}{x}$	x	$\frac{\sin x}{x}$	x	$\frac{\sin x}{x}$	x	$\frac{\sin x}{x}$
7,50	0,12506 666	8,00	0,12366 978	8,50	0,09393 966	9,00	0,04579 094	9,50	-0,00791 064
1	535 544	1	332 757	1	311 718	1	472 661	1	895 046
2	563 093	2	297 389	2	228 732	2	366 016	2	998 721
3	589 315	3	260 881	3	145 020	3	259 172	3	-0,01102 077
4	614 209	4	223 239	4	060 590	4	152 139	4	205 108
5	637 778	5	184 470	5	0,08975 452	5	044 927	5	307 801
6	660 023	6	144 580	6	889 617	6	0,03937 548	6	410 150
7	680 944	7	103 576	7	803 094	7	830 013	7	512 144
8	700 544	8	061 465	8	715 894	8	722 332	8	613 773
9	718 824	9	018 254	9	628 026	9	614 516	9	715 030
7,60	0,12735 785	8,10	0,11973 948	8,60	539 501	9,10	0,03506 575	9,60	815 904
1	751 430	1	928 556	1	450 329	1	398 522	1	916 387
2	765 761	2	882 085	2	360 519	2	290 366	2	-0,02016 469
3	778 779	3	834 541	3	270 082	3	182 118	3	116 143
4	790 486	4	785 932	4	179 029	4	073 790	4	215 398
5	800 886	5	736 265	5	087 369	5	0,02965 391	5	314 226
6	809 980	6	685 547	6	0,07995 113	6	856 932	6	412 618
7	817 771	7	633 787	7	902 272	7	748 425	7	510 566
8	824 262	8	580 991	8	808 854	8	639 879	8	608 061
9	829 455	9	527 167	9	714 872	9	531 306	9	705 093
7,70	0,12833 354	8,20	0,11472 324	8,70	620 336	9,20	0,02422 716	9,70	-0,02801 656
1	835 960	1	416 468	1	525 255	1	314 120	1	897 740
2	837 278	2	359 608	2	429 641	2	205 529	2	993 336
3	837 311	3	301 751	3	333 504	3	096 952	3	-0,03088 437
4	836 061	4	242 906	4	236 854	4	0,01988 401	4	183 035
5	833 533	5	183 081	5	139 702	5	879 886	5	277 120
6	829 729	6	122 284	6	042 059	6	771 418	6	370 685
7	824 654	7	060 523	7	0,06943 935	7	663 007	7	463 722
8	818 311	8	0,10997 806	8	845 342	8	554 663	8	556 222
9	810 704	9	934 143	9	746 288	9	446 397	9	648 179
7,80	0,12801 838	8,30	0,10869 540	8,80	646 786	9,30	0,01338 220	9,80	739 583
1	791 715	1	804 008	1	546 846	1	230 141	1	830 427
2	780 341	2	737 553	2	446 479	2	122 171	2	920 704
3	767 720	3	670 186	3	345 695	3	014 321	3	-0,04010 405
4	753 855	4	601 915	4	244 505	4	0,00906 600	4	099 523
5	738 753	5	532 748	5	142 920	5	799 019	5	188 051
6	722 416	6	462 695	6	040 951	6	691 588	6	275 981
7	704 850	7	391 765	7	0,05938 609	7	584 318	7	363 306
8	686 060	8	319 965	8	835 903	8	477 217	8	450 018
9	666 051	9	247 306	9	732 846	9	370 298	9	536 110
7,90	0,12644 827	8,40	0,10173 797	8,90	629 448	9,40	0,00263 568	9,90	-0,04621 575
1	622 394	1	099 446	1	525 719	1	157 040	1	706 405
2	598 757	2	024 263	2	421 672	2 +	050 721	2	790 595
3	573 922	3	0,09948 257	3	317 315	3	-0,00055 377	3	874 136
4	547 893	4	871 437	4	212 661	4	161 244	4	957 023
5	520 676	5	793 814	5	107 720	5	266 872	5	-0,05039 248
6	492 278	6	715 396	6	002 504	6	372 249	6	120 804
7	462 703	7	636 192	7	0,04897 022	7	477 367	7	201 685
8	431 957	8	556 213	8	791 286	8	582 215	8	281 884
9	400 047	9	475 468	9	685 306	9	686 784	9	361 395

TABLE VI. (CONTINUED)

$\frac{\sin x}{x}$ in Terms of X Radians

x	$\frac{\sin x}{x}$	x	$\frac{\sin x}{x}$
10,0	-0,05440 211	13,0	3232 054
1	6188 818	1	3882 912
2	6861 517	2	4485 405
3	7453 260	3	5034 359
4	7959 870	4	5525 193
5	8378 055	5	5953 959
6	8705 429	6	6317 366
7	8940 514	7	6612 808
8	9082 743	8	6838 374
9	9132 443	9	6992 860
11,0	9090 820	14,0	7075 767
1	8959 933	1	7087 301
2	8742 658	2	7028 357
3	8442 648	3	6900 503
4	8064 285	4	6705 957
5	7612 628	5	6447 552
6	7093 350	6	6128 707
7	6512 680	7	5753 380
8	5877 331	8	5326 027
9	5194 430	9	4851 553
12,0	4471 441	15,0	+0,04335 252
1	3716 095	1	3782 761
2	2936 306	2	3199 991
3	2140 096	3	2593 076
4	1335 518	4	1968 301
5	- 0530 575	5	1332 048
6	+0, 00266 850	6	0690 729
7	1049 071	7	+ 0050 721
8	1808 671	8	-0,00581 689
9	2538 562	9	1200 368

TABLE VII.

1 - cos Δ sin (ø ± β)

ø ± β / Δ	180 0	178 2	176 4	174 6	172 8	170 10	168 12	166 14	164 16	162 18	160 20	158 22	156 24	154 26	152 28	150 30	ø ± β / Δ
0	1.000	.9651	.9302	.8955	.8608	.8264	.7921	.7581	.7244	.6910	.6580	.6254	.5933	.5616	.5305	.5000	0
2	1.000	.9651	.9303	.8955	.8609	.8265	.7922	.7582	.7245	.6912	.6582	.6256	.5935	.5619	.5308	.5003	2
4	1.000	.9652	.9304	.8957	.8612	.8268	.7926	.7587	.7250	.6917	.6588	.6263	.5943	.5627	.5317	.5012	4
6	1.000	.9653	.9306	.8960	.8616	.8273	.7932	.7594	.7259	.6927	.6599	.6274	.5955	.5640	.5331	.5027	6
8	1.000	.9654	.9309	.8965	.8622	.8280	.7941	.7604	.7270	.6940	.6613	.6290	.5972	.5659	.5351	.5049	8
10	1.000	.9656	.9313	.8971	.8629	.8290	.7952	.7618	.7286	.6957	.6632	.6311	.5994	.5683	.5377	.5076	10
12	1.000	.9559	.9318	.8978	.8639	.8301	.7966	.7634	.7304	.6977	.6655	.6336	.6022	.5712	.5408	.5109	12
14	1.000	.9661	.9323	.8986	.8650	.8315	.7983	.7653	.7326	.7002	.6681	.6365	.6053	.5747	.5445	.5149	14
16	1.000	.9665	.9329	.8995	.8662	.8331	.8001	.7674	.7350	.7030	.6712	.6399	.6090	.5786	.5487	.5194	16
18	1.000	.9668	.9337	.9006	.8676	.8349	.8023	.7699	.7379	.7061	.6747	.6437	.6132	.5831	.5535	.5245	18
20	1.000	.9672	.9345	.9018	.8692	.8368	.8046	.7727	.7410	.7096	.6786	.6480	.6178	.5881	.5588	.5302	20
22	1.000	.9676	.9353	.9031	.8710	.8390	.8072	.7757	.7444	.7135	.6829	.6527	.6229	.5935	.5647	.5364	22
24	1.000	.9681	.9363	.9045	.8729	.8414	.8101	.7790	.7482	.7177	.6875	.6578	.6284	.5995	.5711	.5432	24
26	1.000	.9686	.9373	.9061	.8749	.8439	.8131	.7826	.7523	.7223	.6926	.6633	.6344	.6060	.5780	.5506	26
28	1.000	.9692	.9384	.9077	.8771	.8467	.8164	.7864	.7566	.7272	.6980	.6692	.6409	.6129	.5855	.5585	28
30	1.000	.9698	.9396	.9095	.8795	.8496	.8199	.7905	.7613	.7324	.7038	.6756	.6478	.6204	.5934	.5670	30
32	1.000	.9704	.9408	.9114	.8820	.8527	.8237	.7948	.7662	.7379	.7100	.6823	.6551	.6282	.6019	.5760	32
34	1.000	.9711	.9422	.9133	.8846	.8560	.8276	.7994	.7715	.7438	.7165	.6894	.6628	.6366	.6108	.5855	34
36	1.000	.9718	.9436	.9154	.8874	.8595	.8318	.8043	.7770	.7500	.7233	.6969	.6709	.6454	.6202	.5955	36
38	1.000	.9725	.9450	.9176	.8903	.8632	.8362	.8094	.7828	.7565	.7305	.7048	.6795	.6546	.6301	.6060	38
40	1.000	.9733	.9466	.9199	.8934	.8670	.8407	.8147	.7888	.7633	.7380	.7130	.6884	.6642	.6404	.6170	40
42	1.000	.9741	.9482	.9223	.8966	.8710	.8455	.8202	.7952	.7704	.7458	.7216	.6977	.6743	.6511	.6284	42
44	1.000	.9749	.9498	.9248	.8999	.8751	.8504	.8260	.8017	.7777	.7540	.7305	.7074	.6847	.6623	.6403	44
46	1.000	.9758	.9515	.9274	.9033	.8794	.8556	.8319	.8085	.7853	.7624	.7398	.7175	.6955	.6739	.6527	46
48	1.000	.9766	.9533	.9301	.9069	.8838	.8609	.8381	.8156	.7932	.7711	.7493	.7278	.7067	.6859	.6654	48
50	1.000	.9776	.9552	.9328	.9105	.8884	.8664	.8445	.8228	.8014	.7802	.7592	.7386	.7182	.6982	.6786	50
52	1.000	.9785	.9571	.9356	.9143	.8931	.8720	.8511	.8303	.8098	.7894	.7694	.7496	.7301	.7110	.6922	52
54	1.000	.9795	.9590	.9386	.9182	.8979	.8778	.8578	.8380	.8184	.7990	.7798	.7609	.7423	.7241	.7061	54
56	1.000	.9805	.9610	.9415	.9222	.9029	.8837	.8647	.8459	.8272	.8087	.7905	.7726	.7549	.7375	.7204	56
58	1.000	.9815	.9630	.9446	.9262	.9080	.8898	.8718	.8539	.8362	.8188	.8015	.7845	.7677	.7512	.7350	58
60	1.000	.9826	.9651	.9477	.9304	.9132	.8960	.8790	.8622	.8455	.8290	.8127	.7966	.7808	.7653	.7500	60
62	1.000	.9836	.9673	.9509	.9347	.9185	.9024	.8864	.8706	.8549	.8394	.8241	.8090	.7942	.7796	.7653	62
64	1.000	.9847	.9694	.9542	.9390	.9239	.9089	.8939	.8792	.8645	.8501	.8358	.8217	.8078	.7942	.7808	64
66	1.000	.9858	.9716	.9575	.9434	.9294	.9154	.9016	.8879	.8743	.8609	.8476	.8346	.8217	.8090	.7966	66
68	1.000	.9869	.9739	.9608	.9479	.9350	.9221	.9094	.8967	.8842	.8719	.8597	.8476	.8358	.8241	.8127	68
70	1.000	.9881	.9761	.9642	.9524	.9406	.9289	.9173	.9057	.8943	.8830	.8719	.8609	.8501	.8394	.8290	70
72	1.000	.9892	.9784	.9677	.9570	.9463	.9358	.9252	.9148	.9045	.8943	.8842	.8743	.8645	.8549	.8455	72
74	1.000	.9904	.9808	.9712	.9616	.9521	.9427	.9333	.9240	.9148	.9057	.8967	.8879	.8792	.8706	.8622	74
76	1.000	.9916	.9831	.9747	.9663	.9580	.9497	.9415	.9333	.9252	.9173	.9094	.9016	.8939	.8864	.8790	76
78	1.000	.9927	.9855	.9783	.9711	.9639	.9568	.9497	.9427	.9358	.9289	.9221	.9154	.9089	.9024	.8960	78
80	1.000	.9939	.9789	.9818	.9758	.9698	.9639	.9580	.9521	.9463	.9406	.9350	.9294	.9239	.9185	.9132	80
82	1.000	.9951	.9903	.9855	.9806	.9758	.9711	.9663	.9616	.9570	.9524	.9479	.9434	.9390	.9347	.9304	82
84	1.000	.9964	.9927	.9891	.9855	.9818	.9783	.9747	.9712	.9677	.9642	.9608	.9575	.9542	.9509	.9477	84
86	1.000	.9976	.9951	.9927	.9903	.9879	.9855	.9831	.9808	.9784	.9761	.9739	.9716	.9694	.9673	.9651	86
88	1.000	.9988	.9976	.9964	.9951	.9939	.9927	.9916	.9904	.9892	.9881	.9869	.9858	.9847	.9836	.9826	88
90	1.000	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	90
Δ / ø ± β	180 0	178 2	176 4	174 6	172 8	170 10	168 12	166 14	164 16	162 18	160 20	158 22	156 24	154 26	152 28	150 30	Δ / ø ± β

Although the function is tabulated only for positive values of (ø ± β) between 0 and 180°, negative values may be found by disregarding the minus sign and entering the tables with (180-Δ) in place of Δ. When (ø ± β) excedes 180°, add -360° and follow procedure for negative angles as above.

TABLE VII. (CONTINUED)

1 - COS Δ SIN (Ø ± β)

Ø ± β / Δ	148 32	146 34	144 36	142 38	140 40	138 42	136 44	134 46	132 48	130 50	128 52	126 54	124 56	122 58	120 60	Ø ± β / Δ
0	.4701	.4408	.4122	.3843	.3572	.3309	.3053	.2807	.2569	.2340	.2120	.1910	.1710	.1520	.1340	0
2	.4704	.4411	.4126	.3847	.3576	.3313	.3058	.2811	.2573	.2344	.2125	.1915	.1715	.1525	.1345	2
4	.4714	.4422	.4136	.3858	.3588	.3325	.3070	.2824	.2587	.2358	.2139	.1930	.1730	.1540	.1361	4
6	.4730	.4439	.4154	.3877	.3607	.3345	.3091	.2846	.2609	.2382	.2163	.1954	.1755	.1566	.1387	6
8	.4752	.4462	.4179	.3903	.3635	.3374	.3121	.2877	.2641	.2414	.2197	.1989	.1790	.1602	.1424	8
10	.4781	.4493	.4211	.3937	.3670	.3410	.3159	.2916	.2681	.2456	.2240	.2033	.1836	.1648	.1471	10
12	.4817	.4530	.4251	.3978	.3713	.3455	.3205	.2964	.2731	.2507	.2292	.2087	.1891	.1705	.1529	12
14	.4858	.4574	.4297	.4026	.3763	.3507	.3260	.3020	.2789	.2567	.2354	.2150	.1956	.1771	.1597	14
16	.4906	.4625	.4350	.4082	.3821	.3568	.3323	.3085	.2856	.2636	.2425	.2223	.2031	.1848	.1675	16
18	.4960	.4682	.4410	.4145	.3887	.3636	.3393	.3159	.2932	.2714	.2506	.2306	.2115	.1935	.1764	18
20	.5020	.4745	.4477	.4215	.3960	.3712	.3472	.3240	.3017	.2802	.2595	.2398	.2210	.2031	.1862	20
22	.5087	.4815	.4550	.4292	.4040	.3796	.3559	.3330	.3110	.2897	.2694	.2499	.2313	.2137	.1970	22
24	.5159	.4892	.4630	.4376	.4128	.3887	.3654	.3429	.3211	.3002	.2801	.2609	.2426	.2253	.2088	24
26	.5237	.4974	.4717	.4466	.4223	.3986	.3756	.3535	.3321	.3115	.2917	.2729	.2549	.2378	.2216	26
28	.5321	.5063	.4810	.4564	.4325	.4092	.3867	.3649	.3438	.3236	.3042	.2857	.2680	.2512	.2353	28
30	.5411	.5157	.4910	.4668	.4433	.4205	.3984	.3770	.3564	.3366	.3176	.2994	.2820	.2656	.2500	30
32	.5506	.5258	.5015	.4779	.4549	.4325	.4109	.3900	.3698	.3504	.3317	.3139	.2969	.2808	.2656	32
34	.5607	.5364	.5127	.4896	.4671	.4453	.4241	.4036	.3839	.3649	.3467	.3293	.3127	.2969	.2820	34
36	.5713	.5476	.5245	.5019	.4800	.4587	.4380	.4180	.3988	.3803	.3625	.3455	.3293	.3139	.2994	36
38	.5824	.5593	.5368	.5149	.4935	.4727	.4526	.4332	.4144	.3963	.3790	.3625	.3467	.3317	.3176	38
40	.5941	.5716	.5497	.5284	.5076	.4874	.4679	.4490	.4307	.4132	.3963	.3803	.3649	.3504	.3366	40
42	.6062	.5844	.5632	.5425	.5223	.5027	.4838	.4654	.4477	.4307	.4144	.3988	.3839	.3698	.3564	42
44	.6188	.5978	.5772	.5571	.5376	.5187	.5003	.4826	.4654	.4490	.4332	.4180	.4036	.3900	.3770	44
46	.6319	.6116	.5917	.5723	.5535	.5352	.5174	.5003	.4838	.4679	.4526	.4380	.4241	.4109	.3984	46
48	.6454	.6258	.6067	.5880	.5699	.5523	.5352	.5187	.5027	.4874	.4727	.4587	.4453	.4325	.4205	48
50	.6594	.6406	.6222	.6043	.5868	.5699	.5535	.5376	.5223	.5076	.4935	.4800	.4671	.4549	.4433	50
52	.6737	.6557	.6381	.6210	.6043	.5880	.5723	.5571	.5425	.5284	.5149	.5019	.4896	.4779	.4668	52
54	.6885	.6713	.6545	.6381	.6222	.6067	.5917	.5772	.5632	.5497	.5368	.5245	.5127	.5015	.4910	54
56	.7037	.6873	.6713	.6557	.6406	.6258	.6116	.5978	.5844	.5716	.5593	.5476	.5364	.5258	.5157	56
58	.7192	.7037	.6886	.6737	.6594	.6454	.6319	.6188	.6062	.5941	.5824	.5713	.5607	.5506	.5411	58
60	.7350	.7204	.7061	.6922	.6786	.6654	.6527	.6403	.6284	.6170	.6060	.5955	.5855	.5760	.5670	60
62	.7512	.7375	.7241	.7110	.6982	.6859	.6739	.6623	.6511	.6404	.6301	.6202	.6108	.6019	.5934	62
64	.7677	.7549	.7423	.7301	.7182	.7067	.6955	.6847	.6742	.6642	.6546	.6454	.6366	.6282	.6204	64
66	.7845	.7726	.7609	.7496	.7386	.7278	.7175	.7074	.6977	.6884	.6795	.6709	.6628	.6551	.6478	66
68	.8015	.7905	.7798	.7694	.7592	.7493	.7398	.7305	.7216	.7130	.7048	.6969	.6894	.6823	.6756	68
70	.8188	.8087	.7990	.7894	.7802	.7711	.7624	.7540	.7458	.7380	.7305	.7233	.7165	.7100	.7038	70
72	.8362	.8272	.8184	.8098	.8014	.7932	.7853	.7777	.7704	.7633	.7565	.7500	.7438	.7379	.7324	72
74	.8539	.8459	.8380	.8303	.8228	.8156	.8085	.8017	.7952	.7888	.7828	.7770	.7715	.7662	.7613	74
76	.8718	.8647	.8578	.8511	.8445	.8381	.8319	.8260	.8202	.8147	.8094	.8043	.7994	.7948	.7905	76
78	.8898	.8837	.8778	.8720	.8664	.8609	.8556	.8504	.8455	.8407	.8362	.8318	.8276	.8237	.8199	78
80	.9080	.9029	.8979	.8931	.8884	.8838	.8794	.8751	.8710	.8670	.8632	.8595	.8560	.8527	.8496	80
82	.9262	.9222	.9182	.9143	.9105	.9069	.9033	.8999	.8966	.8934	.8903	.8874	.8846	.8820	.8795	82
84	.9446	.9415	.9386	.9356	.9328	.9301	.9274	.9248	.9223	.9199	.9176	.9154	.9133	.9114	.9095	84
86	.9630	.9610	.9590	.9571	.9552	.9533	.9515	.9498	.9482	.9466	.9450	.9436	.9422	.9408	.9396	86
88	.9815	.9805	.9795	.9785	.9776	.9766	.9758	.9749	.9741	.9733	.9725	.9718	.9711	.9704	.9698	88
90	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	90
Δ / Ø ± β	148 32	146 34	144 36	142 38	140 40	138 42	136 44	134 46	132 48	130 50	128 52	126 54	124 56	122 58	120 60	Δ / Ø ± β

Although the function is tabulated only for positive values of (Ø ± β) between 0 and 180°, negative values may be found by disregarding the minus sign and entering the tables with (180−Δ) in place of Δ. When (Ø ± β) exceeds 180°, add −360° and follow procedure for negative angles as above.

TABLE VII. (CONTINUED)

$1 - \cos \Delta \sin (\phi \pm \beta)$

$\phi \pm \beta$ / Δ	118 62	116 64	114 66	112 68	110 70	108 72	106 74	104 76	102 78	100 80	98 82	96 84	94 86	92 88	90 90	$\phi \pm \beta$ / Δ
0	.1171	.1012	.0865	.0728	.0603	.0489	.0387	.0297	.0219	.0152	.0097	.0055	.0024	.0006	0	0
2	.1176	.1018	.0870	.0734	.0609	.0495	.0393	.0303	.0224	.0158	.0103	.0061	.0030	.0012	.0006	2
4	.1192	.1034	.0887	.0751	.0626	.0513	.0411	.0321	.0242	.0176	.0121	.0079	.0049	.0030	.0024	4
6	.1219	.1061	.0915	.0779	.0655	.0542	.0440	.0350	.0272	.0206	.0152	.0109	.0079	.0061	.0055	6
8	.1256	.1100	.0953	.0818	.0695	.0582	.0481	.0391	.0314	.0248	.0194	.0152	.0121	.0103	.0097	8
10	.1305	.1149	.1003	.0869	.0746	.0634	.0533	.0444	.0367	.0302	.0248	.0206	.0176	.0158	.0152	10
12	.1363	.1208	.1064	.0931	.0808	.0697	.0597	.0509	.0432	.0367	.0314	.0272	.0242	.0224	.0219	12
14	.1433	.1279	.1136	.1004	.0882	.0772	.0673	.0585	.0509	.0444	.0391	.0350	.0321	.0303	.0297	14
16	.1513	.1360	.1218	.1087	.0967	.0858	.0760	.0673	.0597	.0533	.0481	.0440	.0411	.0393	.0387	16
18	.1603	.1452	.1312	.1182	.1063	.0955	.0858	.0772	.0697	.0634	.0582	.0542	.0513	.0495	.0489	18
20	.1703	.1554	.1415	.1287	.1170	.1063	.0967	.0882	.0808	.0746	.0695	.0655	.0626	.0609	.0603	20
22	.1813	.1667	.1530	.1403	.1287	.1182	.1087	.1004	.0931	.0869	.0818	.0779	.0751	.0734	.0728	22
24	.1934	.1789	.1654	.1530	.1415	.1312	.1218	.1136	.1064	.1003	.0953	.0915	.0887	.0870	.0865	24
26	.2064	.1922	.1789	.1667	.1554	.1452	.1360	.1279	.1208	.1149	.1100	.1061	.1034	.1018	.1012	26
28	.2204	.2064	.1934	.1813	.1703	.1603	.1513	.1433	.1363	.1305	.1256	.1219	.1192	.1176	.1171	28
30	.2353	.2216	.2088	.1970	.1862	.1764	.1675	.1597	.1529	.1471	.1424	.1387	.1361	.1345	.1340	30
32	.2512	.2378	.2253	.2137	.2031	.1935	.1848	.1771	.1705	.1648	.1602	.1566	.1540	.1525	.1520	32
34	.2680	.2549	.2426	.2313	.2210	.2115	.2031	.1956	.1891	.1836	.1790	.1755	.1730	.1715	.1710	34
36	.2857	.2729	.2609	.2499	.2398	.2306	.2223	.2150	.2087	.2033	.1989	.1954	.1930	.1915	.1910	36
38	.3042	.2917	.2801	.2694	.2595	.2506	.2425	.2354	.2292	.2240	.2197	.2163	.2139	.2125	.2120	38
40	.3236	.3115	.3002	.2897	.2802	.2714	.2636	.2567	.2507	.2456	.2414	.2382	.2358	.2344	.2340	40
42	.3438	.3321	.3211	.3110	.3017	.2932	.2856	.2789	.2731	.2681	.2641	.2609	.2587	.2573	.2569	42
44	.3649	.3535	.3429	.3330	.3240	.3159	.3085	.3020	.2964	.2916	.2877	.2846	.2824	.2811	.2807	44
46	.3867	.3756	.3654	.3559	.3472	.3393	.3323	.3260	.3205	.3159	.3121	.3091	.3070	.3058	.3053	46
48	.4092	.3986	.3887	.3796	.3712	.3636	.3568	.3507	.3455	.3410	.3374	.3345	.3325	.3313	.3309	48
50	.4325	.4223	.4128	.4040	.3960	.3887	.3821	.3763	.3713	.3670	.3635	.3607	.3588	.3576	.3572	50
52	.4564	.4466	.4376	.4292	.4215	.4145	.4082	.4026	.3978	.3937	.3903	.3877	.3858	.3847	.3843	52
54	.4810	.4717	.4630	.4550	.4477	.4410	.4350	.4297	.4251	.4211	.4179	.4154	.4136	.4126	.4122	54
56	.5063	.4974	.4892	.4815	.4745	.4682	.4625	.4574	.4530	.4493	.4462	.4439	.4422	.4411	.4408	56
58	.5321	.5237	.5159	.5087	.5020	.4960	.4906	.4858	.4817	.4781	.4752	.4730	.4714	.4704	.4701	58
60	.5585	.5506	.5432	.5364	.5302	.5245	.5194	.5149	.5109	.5076	.5049	.5027	.5012	.5003	.5000	60
62	.5855	.5780	.5711	.5647	.5588	.5535	.5487	.5445	.5408	.5377	.5351	.5331	.5317	.5308	.5305	62
64	.6129	.6060	.5995	.5935	.5881	.5831	.5786	.5747	.5712	.5683	.5659	.5640	.5627	.5619	.5616	64
66	.6409	.6344	.6284	.6229	.6178	.6132	.6090	.6053	.6022	.5994	.5972	.5955	.5943	.5935	.5933	66
68	.6692	.6633	.6578	.6527	.6480	.6437	.6399	.6365	.6336	.6311	.6290	.6274	.6263	.6256	.6254	68
70	.6980	.6926	.6875	.6829	.6786	.6747	.6712	.6681	.6655	.6632	.6613	.6599	.6588	.6582	.6580	70
72	.7272	.7223	.7177	.7135	.7096	.7061	.7030	.7002	.6977	.6957	.6940	.6927	.6917	.6912	.6910	72
74	.7566	.7523	.7482	.7444	.7410	.7379	.7350	.7326	.7304	.7286	.7270	.7259	.7250	.7245	.7244	74
76	.7864	.7826	.7790	.7757	.7727	.7699	.7674	.7653	.7634	.7618	.7604	.7594	.7587	.7582	.7581	76
78	.8164	.8131	.8101	.8072	.8046	.8023	.8001	.7983	.7966	.7952	.7941	.7932	.7926	.7922	.7921	78
80	.8467	.8439	.8414	.8390	.8368	.8349	.8331	.8315	.8301	.8290	.8280	.8273	.8268	.8265	.8264	80
82	.8771	.8749	.8729	.8710	.8692	.8676	.8662	.8650	.8639	.8629	.8622	.8616	.8612	.8609	.8608	82
84	.9077	.9061	.9045	.9031	.9018	.9006	.8995	.8986	.8978	.8971	.8965	.8960	.8957	.8955	.8955	84
86	.9384	.9373	.9363	.9353	.9345	.9337	.9329	.9323	.9318	.9313	.9309	.9306	.9304	.9303	.9302	86
88	.9692	.9686	.9681	.9676	.9672	.9668	.9665	.9661	.9659	.9656	.9654	.9653	.9652	.9651	.9651	88
90	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	90
Δ / $\phi \pm \beta$	118 62	116 64	114 66	112 68	110 70	108 72	106 74	104 76	102 78	100 80	98 82	96 84	94 86	92 88	90 90	Δ / $\phi \pm \beta$

Although the function is tabulated only for positive values of $(\phi \pm \beta)$ between 0 and 180°, negative values may be found by disregarding the minus sign and entering the tables with $(180-\Delta)$ in place of Δ. When $(\phi \pm \beta)$ exceeds 180°, add -360° and follow procedure for negative angles as above.

TABLE VII. (CONTINUED)

$1 - \cos \Delta \sin (\phi \pm \beta)$

$\phi \pm \beta$ / Δ	180 / 0	178 / 2	176 / 4	174 / 6	172 / 8	170 / 10	168 / 12	166 / 14	164 / 16	162 / 18	160 / 20	158 / 22	156 / 24	154 / 26	152 / 28	150 / 30	$\phi \pm \beta$ / Δ
90	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	90
92	1.0	1.0012	1.0024	1.0036	1.0049	1.0061	1.0073	1.0084	1.0096	1.0108	1.0119	1.0131	1.0142	1.0153	1.0164	1.0174	92
94	1.0	1.0024	1.0049	1.0073	1.0097	1.0121	1.0145	1.0169	1.0192	1.0216	1.0239	1.0261	1.0284	1.0306	1.0327	1.0349	94
96	1.0	1.0036	1.0073	1.0109	1.0145	1.0182	1.0217	1.0253	1.0288	1.0323	1.0358	1.0392	1.0425	1.0458	1.0491	1.0523	96
98	1.0	1.0049	1.0097	1.0145	1.0194	1.0242	1.0289	1.0337	1.0384	1.0430	1.0476	1.0521	1.0566	1.0610	1.0653	1.0696	98
100	1.0	1.0061	1.0121	1.0182	1.0242	1.0302	1.0361	1.0420	1.0479	1.0537	1.0594	1.0650	1.0706	1.0761	1.0815	1.0868	100
102	1.0	1.0073	1.0145	1.0217	1.0289	1.0361	1.0432	1.0503	1.0573	1.0642	1.0711	1.0779	1.0846	1.0911	1.0976	1.1040	102
104	1.0	1.0084	1.0169	1.0253	1.0337	1.0420	1.0503	1.0585	1.0667	1.0748	1.0827	1.0906	1.0984	1.1061	1.1136	1.1210	104
106	1.0	1.0096	1.0192	1.0288	1.0384	1.0479	1.0573	1.0667	1.0760	1.0852	1.0943	1.1033	1.1121	1.1208	1.1294	1.1378	106
108	1.0	1.0108	1.0216	1.0323	1.0430	1.0537	1.0642	1.0748	1.0852	1.0955	1.1057	1.1158	1.1257	1.1355	1.1451	1.1545	108
110	1.0	1.0119	1.0239	1.0358	1.0476	1.0594	1.0711	1.0827	1.0943	1.1057	1.1170	1.1281	1.1391	1.1499	1.1606	1.1710	110
112	1.0	1.0131	1.0261	1.0392	1.0521	1.0650	1.0779	1.0906	1.1033	1.1158	1.1281	1.1403	1.1524	1.1642	1.1759	1.1873	112
114	1.0	1.0142	1.0284	1.0425	1.0566	1.0706	1.0846	1.0984	1.1121	1.1257	1.1391	1.1524	1.1654	1.1783	1.1910	1.2034	114
116	1.0	1.0153	1.0306	1.0458	1.0610	1.0761	1.0911	1.1061	1.1208	1.1355	1.1499	1.1642	1.1783	1.1922	1.2058	1.2192	116
118	1.0	1.0164	1.0327	1.0491	1.0653	1.0815	1.0976	1.1136	1.1294	1.1451	1.1606	1.1759	1.1910	1.2058	1.2204	1.2347	118
120	1.0	1.0174	1.0349	1.0523	1.0696	1.0868	1.1040	1.1210	1.1378	1.1545	1.1710	1.1873	1.2034	1.2192	1.2347	1.2500	120
122	1.0	1.0185	1.0370	1.0554	1.0738	1.0920	1.1102	1.1282	1.1461	1.1638	1.1812	1.1985	1.2155	1.2323	1.2488	1.2650	122
124	1.0	1.0195	1.0390	1.0585	1.0778	1.0971	1.1163	1.1353	1.1541	1.1728	1.1913	1.2095	1.2274	1.2451	1.2625	1.2796	124
126	1.0	1.0205	1.0410	1.0614	1.0818	1.1021	1.1222	1.1422	1.1620	1.1816	1.2010	1.2202	1.2391	1.2577	1.2759	1.2939	126
128	1.0	1.0215	1.0429	1.0644	1.0857	1.1069	1.1280	1.1489	1.1697	1.1902	1.2106	1.2306	1.2504	1.2699	1.2890	1.3078	128
130	1.0	1.0224	1.0448	1.0672	1.0895	1.1116	1.1336	1.1555	1.1772	1.1986	1.2198	1.2408	1.2614	1.2818	1.3018	1.3214	130
132	1.0	1.0234	1.0467	1.0699	1.0931	1.1162	1.1391	1.1619	1.1844	1.2068	1.2289	1.2507	1.2722	1.2933	1.3141	1.3346	132
134	1.0	1.0242	1.0485	1.0726	1.0967	1.1206	1.1444	1.1681	1.1915	1.2147	1.2376	1.2602	1.2825	1.3045	1.3261	1.3473	134
136	1.0	1.0251	1.0502	1.0752	1.1001	1.1249	1.1496	1.1740	1.1983	1.2223	1.2460	1.2695	1.2926	1.3153	1.3377	1.3597	136
138	1.0	1.0259	1.0518	1.0777	1.1034	1.1290	1.1545	1.1798	1.2048	1.2296	1.2542	1.2784	1.3023	1.3258	1.3489	1.3716	138
140	1.0	1.0267	1.0534	1.0801	1.1066	1.1330	1.1593	1.1853	1.2112	1.2367	1.2620	1.2870	1.3116	1.3358	1.3596	1.3830	140
142	1.0	1.0275	1.0550	1.0824	1.1097	1.1368	1.1638	1.1906	1.2172	1.2435	1.2695	1.2952	1.3205	1.3454	1.3699	1.3940	142
144	1.0	1.0282	1.0564	1.0846	1.1126	1.1405	1.1682	1.1957	1.2230	1.2500	1.2767	1.3031	1.3291	1.3546	1.3798	1.4045	144
146	1.0	1.0289	1.0578	1.0867	1.1154	1.1440	1.1724	1.2006	1.2285	1.2562	1.2835	1.3106	1.3372	1.3634	1.3892	1.4145	146
148	1.0	1.0296	1.0592	1.0886	1.1180	1.1473	1.1763	1.2052	1.2338	1.2621	1.2900	1.3177	1.3449	1.3718	1.3981	1.4240	148
150	1.0	1.0302	1.0604	1.0905	1.1205	1.1504	1.1801	1.2095	1.2387	1.2676	1.2962	1.3244	1.3522	1.3796	1.4066	1.4330	150
152	1.0	1.0308	1.0616	1.0923	1.1229	1.1533	1.1836	1.2136	1.2434	1.2728	1.3020	1.3308	1.3591	1.3871	1.4145	1.4415	152
154	1.0	1.0314	1.0627	1.0939	1.1251	1.1561	1.1869	1.2174	1.2477	1.2777	1.3074	1.3367	1.3656	1.3940	1.4220	1.4494	154
156	1.0	1.0319	1.0637	1.0955	1.1271	1.1586	1.1899	1.2210	1.2518	1.2823	1.3125	1.3422	1.3716	1.4005	1.4289	1.4568	156
158	1.0	1.0324	1.0647	1.0969	1.1290	1.1610	1.1928	1.2243	1.2556	1.2865	1.3171	1.3473	1.3771	1.4065	1.4353	1.4636	158
160	1.0	1.0328	1.0655	1.0982	1.1308	1.1632	1.1954	1.2273	1.2590	1.2904	1.3214	1.3520	1.3822	1.4119	1.4412	1.4698	160
162	1.0	1.0332	1.0663	1.0994	1.1324	1.1651	1.1977	1.2301	1.2621	1.2939	1.3253	1.3563	1.3868	1.4169	1.4465	1.4755	162
164	1.0	1.0335	1.0671	1.1005	1.1338	1.1669	1.1999	1.2326	1.2650	1.2970	1.3288	1.3601	1.3910	1.4214	1.4513	1.4806	164
166	1.0	1.0339	1.0677	1.1014	1.1350	1.1685	1.2017	1.2347	1.2674	1.2998	1.3319	1.3635	1.3947	1.4253	1.4555	1.4851	166
168	1.0	1.0341	1.0682	1.1022	1.1361	1.1699	1.2034	1.2366	1.2696	1.3023	1.3345	1.3664	1.3978	1.4288	1.4592	1.4891	168
170	1.0	1.0344	1.0687	1.1029	1.1371	1.1710	1.2048	1.2382	1.2714	1.3043	1.3368	1.3689	1.4006	1.4317	1.4623	1.4924	170
172	1.0	1.0346	1.0691	1.1035	1.1378	1.1720	1.2059	1.2396	1.2730	1.3060	1.3387	1.3710	1.4028	1.4341	1.4649	1.4951	172
174	1.0	1.0347	1.0694	1.1040	1.1384	1.1727	1.2068	1.2406	1.2741	1.3073	1.3401	1.3726	1.4045	1.4360	1.4669	1.4973	174
176	1.0	1.0348	1.0696	1.1043	1.1388	1.1732	1.2074	1.2413	1.2750	1.3083	1.3412	1.3737	1.4057	1.4373	1.4683	1.4988	176
178	1.0	1.0349	1.0697	1.1045	1.1391	1.1735	1.2078	1.2418	1.2755	1.3088	1.3418	1.3744	1.4065	1.4381	1.4692	1.4997	178
180	1.0	1.0349	1.0698	1.1045	1.1392	1.1736	1.2079	1.2419	1.2756	1.3090	1.3420	1.3746	1.4067	1.4384	1.4695	1.5000	180
Δ / $\phi \pm \beta$	180 / 0	178 / 2	176 / 4	174 / 6	172 / 8	170 / 10	168 / 12	166 / 14	164 / 16	162 / 18	160 / 20	158 / 22	156 / 24	154 / 26	152 / 28	150 / 30	Δ / $\phi \pm \beta$

Although the function is tabulated only for positive values of $(\phi \pm \beta)$ between 0 and 180°, negative values may be found by disregarding the minus sign and entering the tables with (180−Δ) in place of Δ. When $(\phi \pm \beta)$ exceeds 180°, add −360° and follow procedure for negative angles as above.

TABLE VII. (CONTINUED)

1 - COS Δ SIN (∅ ± β)

∅ ± β / Δ	148 / 32	146 / 34	144 / 36	142 / 38	140 / 40	138 / 42	136 / 44	134 / 46	132 / 48	130 / 50	128 / 52	126 / 54	124 / 56	122 / 58	120 / 60	∅ ± β / Δ
90	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	90
92	1.0185	1.0195	1.0205	1.0215	1.0224	1.0234	1.0242	1.0251	1.0259	1.0267	1.0275	1.0282	1.0289	1.0296	1.0302	92
94	1.0370	1.0390	1.0410	1.0429	1.0448	1.0467	1.0485	1.0502	1.0518	1.0534	1.0550	1.0564	1.0578	1.0592	1.0604	94
96	1.0554	1.0585	1.0614	1.0644	1.0672	1.0699	1.0726	1.0752	1.0777	1.0801	1.0824	1.0846	1.0867	1.0886	1.0905	96
98	1.0738	1.0778	1.0818	1.0857	1.0895	1.0931	1.0967	1.1001	1.1034	1.1066	1.1097	1.1126	1.1154	1.1180	1.1205	98
100	1.0920	1.0971	1.1021	1.1069	1.1116	1.1162	1.1206	1.1249	1.1290	1.1330	1.1368	1.1405	1.1440	1.1473	1.1504	100
102	1.1102	1.1163	1.1222	1.1280	1.1336	1.1391	1.1444	1.1496	1.1545	1.1593	1.1638	1.1682	1.1724	1.1763	1.1801	102
104	1.1282	1.1353	1.1422	1.1489	1.1555	1.1619	1.1681	1.1740	1.1798	1.1853	1.1906	1.1957	1.2006	1.2052	1.2095	104
106	1.1461	1.1541	1.1620	1.1697	1.1772	1.1844	1.1915	1.1983	1.2048	1.2112	1.2172	1.2230	1.2285	1.2338	1.2387	106
108	1.1638	1.1728	1.1816	1.1902	1.1986	1.2068	1.2147	1.2223	1.2296	1.2367	1.2435	1.2500	1.2562	1.2621	1.2676	108
110	1.1812	1.1913	1.2010	1.2106	1.2198	1.2289	1.2376	1.2460	1.2542	1.2620	1.2695	1.2767	1.2835	1.2900	1.2962	110
112	1.1985	1.2095	1.2202	1.2306	1.2408	1.2507	1.2602	1.2695	1.2784	1.2870	1.2952	1.3031	1.3106	1.3177	1.3244	112
114	1.2155	1.2274	1.2391	1.2504	1.2614	1.2722	1.2825	1.2926	1.3023	1.3116	1.3205	1.3291	1.3372	1.3449	1.3522	114
116	1.2323	1.2451	1.2577	1.2699	1.2818	1.2933	1.3045	1.3153	1.3258	1.3358	1.3454	1.3546	1.3634	1.3718	1.3796	116
118	1.2488	1.2625	1.2759	1.2890	1.3018	1.3141	1.3261	1.3377	1.3489	1.3596	1.3699	1.3798	1.3892	1.3981	1.4066	118
120	1.2650	1.2796	1.2939	1.3078	1.3214	1.3346	1.3473	1.3597	1.3716	1.3830	1.3940	1.4045	1.4145	1.4240	1.4330	120
122	1.2808	1.2963	1.3115	1.3263	1.3406	1.3546	1.3681	1.3812	1.3938	1.4059	1.4176	1.4287	1.4393	1.4494	1.4589	122
124	1.2963	1.3127	1.3287	1.3443	1.3594	1.3742	1.3884	1.4022	1.4156	1.4284	1.4406	1.4524	1.4636	1.4742	1.4843	124
126	1.3115	1.3287	1.3455	1.3619	1.3778	1.3933	1.4083	1.4228	1.4368	1.4503	1.4632	1.4755	1.4873	1.4985	1.5090	126
128	1.3263	1.3443	1.3619	1.3790	1.3957	1.4120	1.4277	1.4429	1.4575	1.4716	1.4851	1.4981	1.5104	1.5221	1.5332	128
130	1.3406	1.3594	1.3778	1.3957	1.4132	1.4301	1.4465	1.4624	1.4777	1.4924	1.5065	1.5200	1.5329	1.5451	1.5567	130
132	1.3546	1.3742	1.3933	1.4120	1.4301	1.4477	1.4648	1.4813	1.4973	1.5126	1.5273	1.5413	1.5547	1.5675	1.5795	132
134	1.3681	1.3884	1.4083	1.4277	1.4465	1.4648	1.4826	1.4997	1.5162	1.5321	1.5474	1.5620	1.5759	1.5891	1.6016	134
136	1.3812	1.4022	1.4228	1.4429	1.4624	1.4813	1.4997	1.5174	1.5346	1.5510	1.5668	1.5820	1.5964	1.6100	1.6230	136
138	1.3938	1.4156	1.4368	1.4575	1.4777	1.4973	1.5162	1.5346	1.5523	1.5693	1.5856	1.6012	1.6161	1.6302	1.6436	138
140	1.4059	1.4284	1.4503	1.4716	1.4924	1.5126	1.5321	1.5510	1.5693	1.5868	1.6037	1.6197	1.6351	1.6496	1.6634	140
142	1.4176	1.4406	1.4632	1.4851	1.5065	1.5273	1.5474	1.5668	1.5856	1.6037	1.6210	1.6375	1.6533	1.6683	1.6824	142
144	1.4287	1.4524	1.4755	1.4981	1.5200	1.5413	1.5620	1.5820	1.6012	1.6197	1.6375	1.6545	1.6707	1.6861	1.7006	144
146	1.4393	1.4636	1.4873	1.5104	1.5329	1.5547	1.5759	1.5964	1.6161	1.6351	1.6533	1.6707	1.6873	1.7031	1.7180	146
148	1.4494	1.4742	1.4985	1.5221	1.5451	1.5675	1.5891	1.6100	1.6302	1.6496	1.6683	1.6861	1.7031	1.7192	1.7344	148
150	1.4589	1.4843	1.5090	1.5332	1.5567	1. 795	1.6016	1.6230	1.6436	1.6634	1.6824	1.7006	1.7180	1.7344	1.7500	150
152	1.4679	1.4937	1.5190	1.5436	1.5675	1.5908	1.6133	1.6351	1.6562	1.6764	1.6958	1.7143	1.7320	1.7488	1.7647	152
154	1.4763	1.5026	1.5283	1.5534	1.5777	1.6014	1.6244	1.6465	1.6679	1.6885	1.7083	1.7271	1.7451	1.7622	1.7784	154
156	1.4841	1.5108	1.5370	1.5624	1.5872	1.6113	1.6346	1.6571	1.6789	1.6998	1.7199	1.7391	1.7574	1.7747	1.7912	156
158	1.4913	1.5185	1.5450	1.5708	1.5960	1.6204	1.6441	1.6670	1.6890	1.7103	1.7306	1.7501	1.7687	1.7863	1.8030	158
160	1.4980	1.5255	1.5523	1.5785	1.6040	1.6288	1.6528	1.6760	1.6983	1.7198	1.7405	1.7602	1.7790	1.7969	1.8138	160
162	1.5040	1.5318	1.5590	1.5855	1.6113	1.6364	1.6607	1.6841	1.7068	1.7286	1.7494	1.7694	1.7885	1.8065	1.8236	162
164	1.5094	1.5375	1.5650	1.5918	1.6179	.6432	1.6677	1.6915	1.7144	1.7364	1.7575	1.7777	1.7969	1.8152	1.8325	164
166	1.5142	1.5426	1.5703	1.5974	1.6237	1.6493	1.6740	1.6980	1.7211	1.7433	1.7647	1.7850	1.8044	1.8229	1.8403	166
168	1.5183	1.5470	1.5749	1.6022	1.6287	1.6545	1.6795	1.7036	1.7269	1.7493	1.7708	1.7913	1.8109	1.8295	1.8471	168
170	1.5219	1.5507	1.5789	1.6063	1.6330	1.6590	1.6841	1.7084	1.7319	1.7544	1.7760	1.7967	1.8164	1.8352	1.8529	170
172	1.5248	1.5538	1.5821	1.6097	1.6365	1.6626	1.6879	1.7123	1.7359	1.7586	1.7803	1.8011	1.8210	1.8398	1.8576	172
174	1.5270	1.5561	1.5846	1.6123	1.6393	1.6655	1.6909	1.7154	1.7391	1.7618	1.7837	1.8046	1.8245	1.8434	1.8613	174
176	1.5286	1.5578	1.5864	1.6142	1.6412	1.6675	1.6930	1.7176	1.7413	1.7642	1.7861	1.8070	1.8270	1.8460	1.8639	176
178	1.5296	1.5589	1.5874	1.6153	1.6424	1.6687	1.6942	1.7189	1.7427	1.7656	1.7875	1.8085	1.8285	1.8475	1.8655	178
180	1.5299	1.5592	1.5878	1.6157	1.6428	1.6691	1.6947	1.7193	1.7431	1.7660	1.7880	1.8090	1.8290	1.8480	1.8660	180
Δ / ∅ ± β	148 / 32	146 / 34	144 / 36	142 / 38	140 / 40	138 / 42	136 / 44	134 / 46	132 / 48	130 / 50	128 / 52	126 / 54	124 / 56	122 / 58	120 / 60	Δ / ∅ ± β

Although the function is tabulated only for positive values of (∅ ± β) between 0 and 180°, negative values may be found by disregarding the minus sign and entering the tables with (180−Δ) in place of Δ. When (∅ ± β) exceeds 180°, add −360° and follow procedure for negative angles as above.

TABLE VII. (CONTINUED)

1 - COS Δ SIN (ø ± β)

ø ± β / Δ	118 62	116 64	114 66	112 68	110 70	108 72	106 74	104 76	102 78	100 80	98 82	96 84	94 86	92 88	90 90	ø ± β / Δ
90	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	90
92	1.0308	1.0314	1.0319	1.0324	1.0328	1.0332	1.0335	1.0339	1.0341	1.0344	1.0346	1.0347	1.0348	1.0349	1.0349	92
94	1.0616	1.0627	1.0637	1.0647	1.0655	1.0663	1.0671	1.0677	1.0682	1.0687	1.0691	1.0694	1.0696	1.0697	1.0698	94
96	1.0923	1.0939	1.0955	1.0969	1.0982	1.0994	1.1005	1.1014	1.1022	1.1029	1.1035	1.1040	1.1043	1.1045	1.1045	96
98	1.1229	1.1251	1.1271	1.1290	1.1308	1.1324	1.1338	1.1350	1.1361	1.1371	1.1378	1.1384	1.1388	1.1391	1.1392	98
100	1.1533	1.1561	1.1586	1.1610	1.1632	1.1651	1.1669	1.1685	1.1699	1.1710	1.1720	1.1727	1.1732	1.1735	1.1736	100
102	1.1836	1.1869	1.1899	1.1928	1.1954	1.1977	1.1999	1.2017	1.2034	1.2048	1.2059	1.2068	1.2074	1.2078	1.2079	102
104	1.2136	1.2174	1.2210	1.2243	1.2273	1.2301	1.2326	1.2347	1.2366	1.2382	1.2396	1.2406	1.2413	1.2418	1.2419	104
106	1.2434	1.2477	1.2518	1.2556	1.2590	1.2621	1.2650	1.2674	1.2696	1.2714	1.2730	1.2741	1.2750	1.2755	1.2756	106
108	1.2728	1.2777	1.2823	1.2865	1.2904	1.2939	1.2970	1.2998	1.3023	1.3043	1.3060	1.3073	1.3083	1.3088	1.3090	108
110	1.3020	1.3074	1.3125	1.3171	1.3214	1.3253	1.3288	1.3319	1.3345	1.3368	1.3387	1.3401	1.3412	1.3418	1.3420	110
1124	1.3308	1.3367	1.3422	1.3473	1.3520	1.3563	1.3601	1.3635	1.3664	1.3689	1.3710	1.3726	1.3737	1.3744	1.3746	112
114	1.3591	1.3656	1.3716	1.3771	1.3822	1.3868	1.3910	1.3947	1.3978	1.4006	1.4028	1.4045	1.4057	1.4065	1.4067	114
116	1.3871	1.3940	1.4005	1.4065	1.4119	1.4169	1.4214	1.4253	1.4288	1.4317	1.4341	1.4360	1.4373	1.4381	1.4384	116
118	1.4145	1.4220	1.4289	1.4353	1.4412	1.4465	1.4513	1.4555	1.4592	1.4623	1.4649	1.4669	1.4683	1.4692	1.4695	118
120	1.4415	1.4494	1.4568	1.4636	1.4698	1.4755	1.4806	1.4851	1.4891	1.4924	1.4951	1.4973	1.4988	1.4997	1.5000	120
122	1.4679	1.4763	1.4841	1.4913	1.4980	1.5040	1.5094	1.5142	1.5183	1.5219	1.5248	1.5270	1.5286	1.5296	1.5299	122
124	1.4937	1.5026	1.5108	1.5185	1.5255	1.5318	1.5375	1.5426	1.5470	1.5507	1.5538	1.5561	1.5578	1.5589	1.5592	124
126	1.5190	1.5283	1.5370	1.5450	1.5523	1.5590	1.5650	1.5703	1.5749	1.5789	1.5821	1.5846	1.5864	1.5874	1.5878	126
128	1.5436	1.5534	1.5624	1.5708	1.5785	1.5855	1.5918	1.5974	1.6022	1.6063	1.6097	1.6123	1.6142	1.6153	1.6157	128
130	1.5675	1.5777	1.5872	1.5960	1.6040	1.6113	1.6179	1.6237	1.6287	1.6330	1.6365	1.6393	1.6412	1.6424	1.6428	130
132	1.5908	1.6014	1.6113	1.6204	1.6288	1.6364	1.6432	1.6493	1.6545	1.6590	1.6626	1.6655	1.6675	1.6687	1.6691	132
134	1.6133	1.6244	1.6346	1.6441	1.6528	1.6607	1.6677	1.6740	1.6795	1.6841	1.6879	1.6909	1.6930	1.6942	1.6947	134
136	1.6351	1.6465	1.6571	1.6670	1.6760	1.6841	1.6915	1.6980	1.7036	1.7084	1.7123	1.7154	1.7176	1.7189	1.7193	136
138	1.6562	1.6679	1.6789	1.6890	1.6983	1.7068	1.7144	1.7211	1.7269	1.7319	1.7359	1.7391	1.7413	1.7427	1.7431	138
140	1.6764	1.6885	1.6998	1.7103	1.7198	1.7286	1.7364	1.7433	1.7493	1.7544	1.7586	1.7618	1.7642	1.7656	1.7660	140
142	1.6958	1.7083	1.7199	1.7306	1.7405	1.7494	1.7575	1.7646	1.7708	1.7760	1.7803	1.7837	1.7861	1.7875	1.7880	142
144	1.7143	1.7271	1.7391	1.7501	1.7602	1.7694	1.7777	1.7850	1.7913	1.7967	1.8011	1.8046	1.8070	1.8085	1.8090	144
146	1.7320	1.7451	1.7574	1.7687	1.7790	1.7885	1.7969	1.8044	1.8109	1.8164	1.8210	1.8245	1.8270	1.8285	1.8290	146
148	1.7488	1.7622	1.7747	1.7863	1.7969	1.8065	1.8152	1.8229	1.8295	1.8352	1.8398	1.8434	1.8460	1.8475	1.8480	148
150	1.7647	1.7784	1.7912	1.8030	1.8138	1.8236	1.8325	1.8403	1.8471	1.8529	1.8576	1.8613	1.8639	1.8655	1.8660	150
152	1.7796	1.7936	1.8066	1.8187	1.8297	1.8397	1.8487	1.8567	1.8637	1.8695	1.8744	1.8781	1.8808	1.8824	1.8829	152
154	1.7936	1.8078	1.8211	1.8333	1.8446	1.8548	1.8640	1.8721	1.8792	1.8851	1.8900	1.8939	1.8966	1.8982	1.8988	154
156	1.8066	1.8211	1.8346	1.8470	1.8585	1.8688	1.8782	1.8864	1.8936	1.8997	1.9047	1.9085	1.9113	1.9130	1.9135	156
158	1.8187	1.8333	1.8470	1.8597	1.8713	1.8818	1.8913	1.8996	1.9069	1.9131	1.9182	1.9221	1.9249	1.9266	1.9272	158
160	1.8297	1.8446	1.8585	1.8713	1.8830	1.8937	1.9033	1.9118	1.9192	1.9254	1.9305	1.9345	1.9374	1.9391	1.9397	160
162	1.8397	1.8548	1.8688	1.8818	1.8937	1.9045	1.9142	1.9228	1.9303	1.9366	1.9418	1.9458	1.9487	1.9505	1.9511	162
164	1.8487	1.8640	1.8782	1.8913	1.9033	1.9142	1.9240	1.9327	1.9403	1.9467	1.9519	1.9560	1.9589	1.9607	1.9613	164
166	1.8567	1.8721	1.8864	1.8996	1.9118	1.9228	1.9327	1.9415	1.9491	1.9556	1.9609	1.9650	1.9679	1.9697	1.9307	166
168	1.8637	1.8792	1.8936	1.9069	1.9192	1.9303	1.9403	1.9491	1.9568	1.9633	1.9686	1.9728	1.9758	1.9776	1.9781	168
170	1.8695	1.8851	1.8997	1.9131	1.9254	1.9366	1.9467	1.9556	1.9633	1.9698	1.9752	1.9794	1.9824	1.9842	1.9848	170
172	1.8744	1.8900	1.9047	1.9182	1.9305	1.9418	1.9519	1.9609	1.9686	1.9752	1.9806	1.9848	1.9879	1.9897	1.9903	172
174	1.8781	1.8939	1.9085	1.9221	1.9345	1.9458	1.9560	1.9650	1.9728	1.9794	1.9848	1.9891	1.9921	1.9939	1.9945	174
176	1.8808	1.8966	1.9113	1.9249	1.9374	1.9487	1.9589	1.9679	1.9758	1.9824	1.9879	1.9921	1.9951	1.9970	1.9976	176
178	1.8824	1.8982	1.9130	1.9266	1.9391	1.9505	1.9607	1.9697	1.9776	1.9842	1.9897	1.9939	1.9970	1.9988	1.9994	178
180	1.8829	1.8988	1.9135	1.9272	1.9397	1.9511	1.9613	1.9703	1.9781	1.9848	1.9903	1.9945	1.9976	1.9994	2.0000	180
Δ / ø ± β	118 62	116 64	114 66	112 68	110 70	108 72	106 74	104 76	102 78	100 80	98 82	96 84	94 86	92 88	90 90	Δ / ø ± β

Although the function is tabulated only for positive values of (ø ± β) between 0 and 180°, negative values may be found by disregarding the minus sign and entering the tables with (180-Δ) in place of Δ. When (ø ± β) exceeds 180°, add -360° and follow procedure for negative angles as above.

TABLE VIII

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Trigonometric Functions.

RADIANS.	DEGREES.	SINES.		COSINES.		TANGENTS.		COTANGENTS.			
		Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.		
0.0000	0° 00′	.0000	∞	1.0000	0.0000	.0000	∞	∞	∞	90° 00′	1.5708
0.0029	10	.0029	7.4637	1.0000	.0000	.0029	7.4637	343.77	2.5363	50	1.5679
0.0058	20	.0058	.7648	1.0000	.0000	.0058	.7648	171.89	.2352	40	1.5650
0.0087	30	.0087	.9408	1.0000	.0000	.0087	.9409	114.59	.0591	30	1.5621
0.0116	40	.0116	8.0658	.9999	.0000	.0116	8.0658	85.940	1.9342	20	1.5592
0.0145	50	.0145	.1627	.9999	.0000	.0145	.1627	68.750	.8373	10	1.5563
0.0175	1° 00′	.0175	8.2419	.9998	9.9999	.0175	8.2419	57.290	1.7581	89° 00′	1.5533
0.0204	10	.0204	.3088	.9998	.9999	.0204	.3089	49.104	.6911	50	1.5504
0.0233	20	.0233	.3668	.9997	.9999	.0233	.3669	42.964	.6331	40	1.5475
0.0262	30	.0262	.4179	.9997	.9999	.0262	.4181	38.188	.5819	30	1.5446
0.0291	40	.0291	.4637	.9996	.9998	.0291	.4638	34.368	.5362	20	1.5417
0.0320	50	.0320	.5050	.9995	.9998	.0320	.5053	31.242	.4947	10	1.5388
0.0349	2° 00′	.0349	8.5428	.9994	9.9997	.0349	8.5431	28.636	1.4569	88° 00′	1.5359
0.0378	10	.0378	.5776	.9993	.9997	.0378	.5779	26.432	.4221	50	1.5330
0.0407	20	.0407	.6097	.9992	.9996	.0407	.6101	24.542	.3899	40	1.5301
0.0436	30	.0436	.6397	.9990	.9996	.0437	.6401	22.904	.3599	30	1.5272
0.0465	40	.0465	.6677	.9989	.9995	.0466	.6682	21.470	.3318	20	1.5243
0.0495	50	.0494	.6940	.9988	.9995	.0495	.6945	20.206	.3055	10	1.5213
0.0524	3° 00′	.0523	8.7188	.9986	9.9994	.0524	8.7194	19.081	1.2806	87° 00′	1.5184
0.0553	10	.0552	.7423	.9985	.9993	.0553	.7429	18.075	.2571	50	1.5155
0.0582	20	.0581	.7645	.9983	.9993	.0582	.7652	17.169	.2348	40	1.5126
0.0611	30	.0610	.7857	.9981	.9992	.0612	.7865	16.350	.2135	30	1.5097
0.0640	40	.0640	.8059	.9980	.9991	.0641	.8067	15.605	.1933	20	1.5068
0.0669	50	.0669	.8251	.9978	.9990	.0670	.8261	14.924	.1739	10	1.5039
0.0698	4° 00′	.0698	8.8436	.9976	9.9989	.0699	8.8446	14.301	1.1554	86° 00′	1.5010
0.0727	10	.0727	.8613	.9974	.9989	.0729	.8624	13.727	.1376	50	1.4981
0.0756	20	.0756	.8783	.9971	.9988	.0758	.8795	13.197	.1205	40	1.4952
0.0785	30	.0785	.8946	.9969	.9987	.0787	.8960	12.706	.1040	30	1.4923
0.0814	40	.0814	.9104	.9967	.9986	.0816	.9118	12.251	.0882	20	1.4893
0.0844	50	.0843	.9256	.9964	.9985	.0846	.9272	11.826	.0728	10	1.4864
0.0873	5° 00′	.0872	8.9403	.9962	9.9983	.0875	8.9420	11.430	1.0580	85° 00′	1.4835
0.0902	10	.0901	.9545	.9959	.9982	.0904	.9563	11.059	.0437	50	1.4806
0.0931	20	.0929	.9682	.9957	.9981	.0934	.9701	10.712	.0299	40	1.4777
0.0960	30	.0958	.9816	.9954	.9980	.0963	.9836	10.385	.0164	30	1.4748
0.0989	40	.0987	.9945	.9951	.9979	.0992	.9966	10.078	.0034	20	1.4719
0.1018	50	.1016	9.0070	.9948	.9977	.1022	9.0093	9.7882	0.9907	10	1.4690
0.1047	6° 00′	.1045	9.0192	.9945	9.9976	.1051	9.0216	9.5144	0.9784	84° 00′	1.4661
0.1076	10	.1074	.0311	.9942	.9975	.1080	.0336	9.2553	.9664	50	1.4632
0.1105	20	.1103	.0426	.9939	.9973	.1110	.0453	9.0098	.9547	40	1.4603
0.1134	30	.1132	.0539	.9936	.9972	.1139	.0567	8.7769	.9433	30	1.4574
0.1164	40	.1161	.0648	.9932	.9971	.1169	.0678	8.5555	.9322	20	1.4544
0.1193	50	.1190	.0755	.9929	.9969	.1198	.0786	8.3450	.9214	10	1.4515
0.1222	7° 00′	.1219	9.0859	.9925	9.9968	.1228	9.0891	8.1443	0.9109	83° 00′	1.4486
0.1251	10	.1248	.0961	.9922	.9966	.1257	.0995	7.9530	.9005	50	1.4457
0.1280	20	.1276	.1060	.9918	.9964	.1287	.1096	7.7704	.8904	40	1.4428
0.1309	30	.1305	.1157	.9914	.9963	.1317	.1194	7.5958	.8806	30	1.4399
0.1338	40	.1334	.1252	.9911	.9961	.1346	.1291	7.4287	.8709	20	1.4370
0.1367	50	.1363	.1345	.9907	.9959	.1376	.1385	7.2687	.8615	10	1.4341
0.1396	8° 00′	.1392	9.1436	.9903	9.9958	.1405	9.1478	7.1154	0.8522	82° 00′	1.4312
0.1425	10	.1421	.1525	.9899	.9956	.1435	.1569	6.9682	.8431	50	1.4283
0.1454	20	.1449	.1612	.9894	.9954	.1465	.1658	6.8269	.8342	40	1.4254
0.1484	30	.1478	.1697	.9890	.9952	.1495	.1745	6.6912	.8255	30	1.4224
0.1513	40	.1507	.1781	.9886	.9950	.1524	.1831	6.5606	.8169	20	1.4195
0.1542	50	.1536	.1863	.9881	.9948	.1554	.1915	6.4348	.8085	10	1.4166
0.1571	9° 00′	.1564	9.1943	.9877	9.9946	.1584	9.1997	6.3138	0.8003	81° 00′	1.4137
		Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.		
		COSINES.		SINES.		COTANGENTS.		TANGENTS.		DEGREES.	RADIANS.

TABLE VIII (Continued)

Trigonometric Functions.

RADIANS.	DEGREES	SINES. Nat.	SINES. Log.	COSINES. Nat.	COSINES. Log.	TANGENTS. Nat.	TANGENTS. Log.	COTANGENTS. Nat.	COTANGENTS. Log.		
0.1571	9° 00′	.1564	9.1943	.9877	9.9946	.1584	9.1997	6.3138	0.8003	81° 00′	1.4137
0.1600	10	.1593	.2022	.9872	.9944	.1614	.2078	6.1970	.7922	50	1.4108
0.1629	20	.1622	.2100	.9868	.9942	.1644	.2158	6.0844	.7842	40	1.4079
0.1658	30	.1650	.2176	.9863	.9940	.1673	.2236	5.9758	.7764	30	1.4050
0.1687	40	.1679	.2251	.9858	.9938	.1703	.2313	5.8708	.7687	20	1.4021
0.1716	50	.1708	.2324	.9853	.9936	.1733	.2389	5.7694	.7611	10	1.3992
0.1745	10° 00′	.1736	9.2397	.9848	9.9934	.1763	9.2463	5.6713	0.7537	80° 00′	1.3963
0.1774	10	.1765	.2468	.9843	.9931	.1793	.2536	5.5764	.7464	50	1.3934
0.1804	20	.1794	.2538	.9838	.9929	.1823	.2609	5.4845	.7391	40	1.3904
0.1833	30	.1822	.2606	.9833	.9927	.1853	.2680	5.3955	.7320	30	1.3875
0.1862	40	.1851	.2674	.9827	.9924	.1883	.2750	5.3093	.7250	20	1.3846
0.1891	50	.1880	.2740	.9822	.9922	.1914	.2819	5.2257	.7181	10	1.3817
0.1920	11° 00′	.1908	9.2806	.9816	9.9919	.1944	9.2887	5.1446	0.7113	79° 00′	1.3788
0.1949	10	.1937	.2870	.9811	.9917	.1974	.2953	5.0658	.7047	50	1.3759
0.1978	20	.1965	.2934	.9805	.9914	.2004	.3020	4.9894	.6980	40	1.3730
0.2007	30	.1994	.2997	.9799	.9912	.2035	.3085	4.9152	.6915	30	1.3701
0.2036	40	.2022	.3058	.9793	.9909	.2065	.3149	4.8430	.6851	20	1.3672
0.2065	50	.2051	.3119	.9787	.9907	.2095	.3212	4.7729	.6788	10	1.3643
0.2094	12° 00′	.2079	9.3179	.9781	9.9904	.2126	9.3275	4.7046	0.6725	78° 00′	1.3614
0.2123	10	.2108	.3238	.9775	.9901	.2156	.3336	4.6382	.6664	50	1.3584
0.2153	20	.2136	.3296	.9769	.9899	.2186	.3397	4.5736	.6603	40	1.3555
0.2182	30	.2164	.3353	.9763	.9896	.2217	.3458	4.5107	.6542	30	1.3526
0.2211	40	.2193	.3410	.9757	.9893	.2247	.3517	4.4494	.6483	20	1.3497
0.2240	50	.2221	.3466	.9750	.9890	.2278	.3576	4.3897	.6424	10	1.3468
0.2269	13° 00′	.2250	9.3521	.9744	9.9887	.2309	9.3634	4.3315	0.6366	77° 00′	1.3439
0.2298	10	.2278	.3575	.9737	.9884	.2339	.3691	4.2747	.6309	50	1.3410
0.2327	20	.2306	.3629	.9730	.9881	.2370	.3748	4.2193	.6252	40	1.3381
0.2356	30	.2334	.3682	.9724	.9878	.2401	.3804	4.1653	.6196	30	1.3352
0.2385	40	.2363	.3734	.9717	.9875	.2432	.3859	4.1126	.6141	20	1.3323
0.2414	50	.2391	.3786	.9710	.9872	.2462	.3914	4.0611	.6086	10	1.3294
0.2443	14° 00′	.2419	9.3837	.9703	9.9869	.2493	9.3968	4.0108	0.6032	76° 00′	1.3265
0.2473	10	.2447	.3887	.9696	.9866	.2524	.4021	3.9617	.5979	50	1.3235
0.2502	20	.2476	.3937	.9689	.9863	.2555	.4074	3.9136	.5926	40	1.3206
0.2531	30	.2504	.3986	.9681	.9859	.2586	.4127	3.8667	.5873	30	1.3177
0.2560	40	.2532	.4035	.9674	.9856	.2617	.4178	3.8208	.5822	20	1.3148
0.2589	50	.2560	.4083	.9667	.9853	.2648	.4230	3.7760	.5770	10	1.3119
0.2618	15° 00′	.2588	9.4130	.9659	9.9849	.2679	9.4281	3.7321	0.5719	75° 00′	1.3090
0.2647	10	.2616	.4177	.9652	.9846	.2711	.4331	3.6891	.5669	50	1.3061
0.2676	20	.2644	.4223	.9644	.9843	.2742	.4381	3.6470	.5619	40	1.3032
0.2705	30	.2672	.4269	.9636	.9839	.2773	.4430	3.6059	.5570	30	1.3003
0.2734	40	.2700	.4314	.9628	.9836	.2805	.4479	3.5656	.5521	20	1.2974
0.2763	50	.2728	.4359	.9621	.9832	.2836	.4527	3.5261	.5473	10	1.2945
0.2793	16° 00′	.2756	9.4403	.9613	9.9828	.2867	9.4575	3.4874	0.5425	74° 00′	1.2915
0.2822	10	.2784	.4447	.9605	.9825	.2899	.4622	3.4495	.5378	50	1.2886
0.2851	20	.2812	.4491	.9596	.9821	.2931	.4669	3.4124	.5331	40	1.2857
0.2880	30	.2840	.4533	.9588	.9817	.2962	.4716	3.3759	.5284	30	1.2828
0.2909	40	.2868	.4576	.9580	.9814	.2994	.4762	3.3402	.5238	20	1.2799
0.2938	50	.2896	.4618	.9572	.9810	.3026	.4808	3.3052	.5192	10	1.2770
0.2967	17° 00′	.2924	9.4659	.9563	9.9806	.3057	9.4853	3.2709	0.5147	73° 00′	1.2741
0.2996	10	.2952	.4700	.9555	.9802	.3089	.4898	3.2371	.5102	50	1.2712
0.3025	20	.2979	.4741	.9546	.9798	.3121	.4943	3.2041	.5057	40	1.2683
0.3054	30	.3007	.4781	.9537	.9794	.3153	.4987	3.1716	.5013	30	1.2654
0.3083	40	.3035	.4821	.9528	.9790	.3185	.5031	3.1397	.4969	20	1.2625
0.3113	50	.3062	.4861	.9520	.9786	.3217	.5075	3.1084	.4925	10	1.2595
0.3142	18° 00′	.3090	9.4900	.9511	9.9782	.3249	9.5118	3.0777	0.4882	72° 00′	1.2566
		Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.		
		COSINES.		SINES.		COTANGENTS.		TANGENTS.		DEGREES.	RADIANS.

TABLE VIII (Continued)

Trigonometric Functions.

RADIANS.	DEGREES.	SINES.		COSINES.		TANGENTS.		COTANGENTS.			
		Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.		
0.3142	18° 00′	.3090	9.4900	.9511	9.9782	.3249	9.5118	3.0777	0.4882	72° 00′	1.2566
0.3171	10	.3118	.4939	.9502	.9778	.3281	.5161	3.0475	.4839	50	1.2537
0.3200	20	.3145	.4977	.9492	.9774	.3314	.5203	3.0178	.4797	40	1.2508
0.3229	30	.3173	.5015	.9483	.9770	.3346	.5245	2.9887	.4755	30	1.2479
0.3258	40	.3201	.5052	.9474	.9765	.3378	.5287	2.9600	.4713	20	1.2450
0.3287	50	.3228	.5090	.9465	.9761	.3411	.5329	2.9319	.4671	10	1.2421
0.3316	19° 00′	.3256	9.5126	.9455	9.9757	.3443	9.5370	2.9042	0.4630	71° 00′	1.2392
0.3345	10	.3283	.5163	.9446	.9752	.3476	.5411	2.8770	.4589	50	1.2363
0.3374	20	.3311	.5199	.9436	.9748	.3508	.5451	2.8502	.4549	40	1.2334
0.3403	30	.3338	.5235	.9426	.9743	.3541	.5491	2.8239	.4509	30	1.2305
0.3432	40	.3365	.5270	.9417	.9739	.3574	.5531	2.7980	.4469	20	1.2275
0.3462	50	.3393	.5306	.9407	.9734	.3607	.5571	2.7725	.4429	10	1.2246
0.3491	20° 00′	.3420	9.5341	.9397	9.9730	.3640	9.5611	2.7475	0.4389	70° 00′	1.2217
0.3520	10	.3448	.5375	.9387	.9725	.3673	.5650	2.7228	.4350	50	1.2188
0.3549	20	.3475	.5409	.9377	.9721	.3706	.5689	2.6985	.4311	40	1.2159
0.3578	30	.3502	.5443	.9367	.9716	.3739	.5727	2.6746	.4273	30	1.2130
0.3607	40	.3529	.5477	.9356	.9711	.3772	.5766	2.6511	.4234	20	1.2101
0.3636	50	.3557	.5510	.9346	.9706	.3805	.5804	2.6279	.4196	10	1.2072
0.3665	21° 00′	.3584	9.5543	.9336	9.9702	.3839	9.5842	2.6051	0.4158	69° 00′	1.2043
0.3694	10	.3611	.5576	.9325	.9697	.3872	.5879	2.5826	.4121	50	1.2014
0.3723	20	.3638	.5609	.9315	.9692	.3906	.5917	2.5605	.4083	40	1.1985
0.3752	30	.3665	.5641	.9304	.9687	.3939	.5954	2.5386	.4046	30	1.1956
0.3782	40	.3692	.5673	.9293	.9682	.3973	.5991	2.5172	.4009	20	1.1926
0.3811	50	.3719	.5704	.9283	.9677	.4006	.6028	2.4960	.3972	10	1.1897
0.3840	22° 00′	.3746	9.5736	.9272	9.9672	.4040	9.6064	2.4751	0.3936	68° 00′	1.1868
0.3869	10	.3773	.5767	.9261	.9667	.4074	.6100	2.4545	.3900	50	1.1839
0.3898	20	.3800	.5798	.9250	.9661	.4108	.6136	2.4342	.3864	40	1.1810
0.3927	30	.3827	.5828	.9239	.9656	.4142	.6172	2.4142	.3828	30	1.1781
0.3956	40	.3854	.5859	.9228	.9651	.4176	.6208	2.3945	.3792	20	1.1752
0.3985	50	.3881	.5889	.9216	.9646	.4210	.6243	2.3750	.3757	10	1.1723
0.4014	23° 00′	.3907	9.5919	.9205	9.9640	.4245	9.6279	2.3559	0.3721	67° 00′	1.1694
0.4043	10	.3934	.5948	.9194	.9635	.4279	.6314	2.3369	.3686	50	1.1665
0.4072	20	.3961	.5978	.9182	.9629	.4314	.6348	2.3183	.3652	40	1.1636
0.4102	30	.3987	.6007	.9171	.9624	.4348	.6383	2.2998	.3617	30	1.1606
0.4131	40	.4014	.6036	.9159	.9618	.4383	.6417	2.2817	.3583	20	1.1577
0.4160	50	.4041	.6065	.9147	.9613	.4417	.6452	2.2637	.3548	10	1.1548
0.4189	24° 00′	.4067	9.6093	.9135	9.9607	.4452	9.6486	2.2460	0.3514	66° 00′	1.1519
0.4218	10	.4094	.6121	.9124	.9602	.4487	.6520	2.2286	.3480	50	1.1490
0.4247	20	.4120	.6149	.9112	.9596	.4522	.6553	2.2113	.3447	40	1.1461
0.4276	30	.4147	.6177	.9100	.9590	.4557	.6587	2.1943	.3413	30	1.1432
0.4305	40	.4173	.6205	.9088	.9584	.4592	.6620	2.1775	.3380	20	1.1403
0.4334	50	.4200	.6232	.9075	.9579	.4628	.6654	2.1609	.3346	10	1.1374
0.4363	25° 00′	.4226	9.6259	.9063	9.9573	.4663	9.6687	2.1445	0.3313	65° 00′	1.1345
0.4392	10	.4253	.6286	.9051	.9567	.4699	.6720	2.1283	.3280	50	1.1316
0.4422	20	.4279	.6313	.9038	.9561	.4734	.6752	2.1123	.3248	40	1.1286
0.4451	30	.4305	.6340	.9026	.9555	.4770	.6785	2.0965	.3215	30	1.1257
0.4480	40	.4331	.6366	.9013	.9549	.4806	.6817	2.0809	.3183	20	1.1228
0.4509	50	.4358	.6392	.9001	.9543	.4841	.6850	2.0655	.3150	10	1.1199
0.4538	26° 00′	.4384	9.6418	.8988	9.9537	.4877	9.6882	2.0503	0.3118	64° 00′	1.1170
0.4567	10	.4410	.6444	.8975	.9530	.4913	.6914	2.0353	.3086	50	1.1141
0.4596	20	.4436	.6470	.8962	.9524	.4950	.6946	2.0204	.3054	40	1.1112
0.4625	30	.4462	.6495	.8949	.9518	.4986	.6977	2.0057	.3023	30	1.1083
0.4654	40	.4488	.6521	.8936	.9512	.5022	.7009	1.9912	.2991	20	1.1054
0.4683	50	.4514	.6546	.8923	.9505	.5059	.7040	1.9768	.2960	10	1.1025
0.4712	27° 00′	.4540	9.6570	.8910	9.9499	.5095	9.7072	1.9626	0.2928	63° 00′	1.0996
		Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.		
		COSINES.		SINES.		COTANGENTS.		TANGENTS.		DEGREES.	RADIANS.

TABLE VIII (Continued)

Trigonometric Functions.

RADIANS.	DEGREES.	SINES.		COSINES.		TANGENTS.		COTANGENTS.			
		Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.		
0.4712	27° 00′	.4540	9.6570	.8910	9.9499	.5095	9.7072	1.9626	0.2928	63° 00′	1.0996
0.4741	10	.4566	.6595	.8897	.9492	.5132	.7103	1.9486	.2897	50	1.0966
0.4771	20	.4592	.6620	.8884	.9486	.5169	.7134	1.9347	.2866	40	1.0937
0.4800	30	.4617	.6644	.8870	.9479	.5206	,7165	1.9210	.2835	30	1.0908
0.4829	40	.4643	.6668	.8857	.9473	.5243	.7196	1.9074	.2804	20	1.0879
0.4858	50	.4669	.6692	.8843	.9466	.5280	.7226	1.8940	.2774	10	1.0850
0.4887	28° 00′	.4695	9.6716	.8829	9.9459	.5317	9.7257	1.8807	0.2743	62° 00′	1.0821
0.4916	10	.4720	.6740	.8816	.9453	.5354	.7287	1.8676	.2713	50	1.0792
0.4945	20	.4746	.6763	.8802	.9446	.5392	.7317	1.8546	.2683	40	1.0763
0.4974	30	.4772	.6787	.8788	.9439	.5430	.7348	1.8418	.2652	30	1.0734
0.5003	40	.4797	.6810	.8774	.9432	.5467	.7378	1.8291	.2622	20	1.0705
0.5032	50	.4823	.6833	.8760	.9425	.5505	.7408	1.8165	.2592	10	1.0676
0.5061	29° 00′	.4848	9.6856	.8746	9.9418	.5543	9.7438	1.8040	0.2562	61° 00′	1.0647
0.5091	10	.4874	.6878	.8732	.9411	.5581	.7467	1.7917	.2533	50	1.0617
0.5120	20	.4899	.6901	.8718	.9404	.5619	.7497	1.7796	.2503	40	1.0588
0.5149	30	.4924	.6923	.8704	.9397	.5658	.7526	1.7675	.2474	30	1.0559
0.5178	40	.4950	.6946	.8689	.9390	.5696	.7556	1.7556	.2444	20	1.0530
0.5207	50	.4975	.6968	.8675	.9383	.5735	.7585	1.7437	.2415	10	1.0501
0.5236	30° 00′	.5000	9.6990	.8660	9.9375	.5774	9.7614	1.7321	0.2386	60° 00′	1.0472
0.5265	10	.5025	.7012	.8646	.9368	.5812	.7644	1.7205	.2356	50	1.0443
0.5294	20	.5050	.7033	.8631	.9361	.5851	.7673	1.7090	.2327	40	1.0414
0.5323	30	.5075	.7055	.8616	.9353	.5890	.7701	1.6977	.2299	30	1.0385
0.5352	40	.5100	.7076	.8601	.9346	.5930	.7730	1.6864	.2270	20	1.0356
0.5381	50	.5125	.7097	.8587	.9338	.5969	.7759	1.6753	.2241	10	1.0327
0.5411	31° 00′	.5150	9.7118	.8572	9.9331	.6009	9.7788	1.6643	0.2212	59° 00′	1.0297
0.5440	10	.5175	.7139	.8557	.9323	.6048	.7816	1.6534	.2184	50	1.0268
0.5469	20	.5200	.7160	.8542	.9315	.6088	.7845	1.6426	.2155	40	1.0239
0.5498	30	.5225	.7181	.8526	.9308	.6128	.7873	1.6319	.2127	30	1.0210
0.5527	40	.5250	.7201	.8511	.9300	.6168	.7902	1.6212	.2098	20	1.0181
0.5556	50	.5275	.7222	.8496	.9292	.6208	.7930	1.6107	.2070	10	1.0152
0.5585	32° 00′	.5299	9.7242	.8480	9.9284	.6249	9.7958	1.6003	0.2042	58° 00′	1.0123
0.5614	10	.5324	.7262	.8465	.9276	.6289	.7986	1.5900	.2014	50	1.0094
0.5643	20	.5348	.7282	.8450	.9268	.6330	.8014	1.5798	.1986	40	1.0065
0.5672	30	.5373	.7302	.8434	.9260	.6371	.8042	1.5697	.1958	30	1.0036
0.5701	40	.5398	.7322	.8418	.9252	.6412	.8070	1.5597	.1930	20	1.0007
0.5730	50	.5422	.7342	.8403	.9244	.6453	.8097	1.5497	.1903	10	0.9977
0.5760	33° 00′	.5446	9.7361	.8387	9.9236	.6494	9.8125	1.5399	0.1875	57° 00′	0.9948
0.5789	10	.5471	.7380	.8371	.9228	.6536	.8153	1.5301	.1847	50	0.9919
0.5818	20	.5495	.7400	.8355	.9219	.6577	.8180	1.5204	.1820	40	0.9890
0.5847	30	.5519	.7419	.8339	.9211	.6619	.8208	1.5108	.1792	30	0.9861
0.5876	40	.5544	.7438	.8323	.9203	.6661	.8235	1.5013	.1765	20	0.9832
0.5905	50	.5568	.7457	.8307	.9194	.6703	.8263	1.4919	.1737	10	0.9803
0.5934	34° 00′	.5592	9.7476	.8290	9.9186	.6745	9.8290	1.4826	0.1710	56° 00′	0.9774
0.5963	10	.5616	.7494	.8274	.9177	.6787	.8317	1.4733	.1683	50	0.9745
0.5992	20	.5640	.7513	.8258	.9169	.6830	.8344	1.4641	.1656	40	0.9716
0.6021	30	.5664	.7531	.8241	.9160	.6873	.8371	1.4550	.1629	30	0.9687
0.6050	40	.5688	.7550	.8225	.9151	.6916	.8398	1.4460	.1602	20	0.9657
0.6080	50	.5712	.7568	.8208	.9142	.6959	.8425	1.4370	.1575	10	0.9628
0.6109	35° 00′	.5736	9.7586	.8192	9.9134	.7002	9.8452	1.4281	0.1548	55° 00′	0.9599
0.6138	10	.5760	.7604	.8175	.9125	.7046	.8479	1.4193	.1521	50	0.9570
0.6167	20	.5783	.7622	.8158	.9116	.7089	.8506	1.4106	.1494	40	0.9541
0.6196	30	.5807	.7640	.8141	.9107	.7133	.8533	1.4019	.1467	30	0.9512
0.6225	40	.5831	.7657	.8124	.9098	.7177	.8559	1.3934	.1441	20	0.9483
0 6254	50	.5854	7675	.8107	.9089	.7221	.8586	1.3848	.1414	10	0.9454
0.6283	36° 00′	.5878	9.7692	.8090	9.9080	.7265	9.8613	1.3764	0.1387	54° 00′	0.9425
		Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.		
		COSINES.		SINES.		COTANGENTS.		TANGENTS.		DEGREES.	RADIANS.

TABLE VIII (Continued)

Trigonometric Functions.

RADIANS.	DEGREES.	SINES.		COSINES.		TANGENTS.		COTANGENTS.			
		Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.		
0.6283	36° 00′	.5878	9.7692	.8090	9.9080	.7265	9.8613	1.3764	0.1387	54° 00′	0.9425
0.6312	10	.5901	.7710	.8073	.9070	.7310	.8639	1.3680	.1361	50	0.9396
0.6341	20	.5925	.7727	.8056	.9061	.7355	.8666	1.3597	.1334	40	0.9367
0.6370	30	.5948	.7744	.8039	.9052	.7400	.8692	1.3514	.1308	30	0.9338
0.6400	40	.5972	.7761	.8021	.9042	.7445	.8718	1.3432	.1282	20	0.9308
0.6429	50	.5995	.7778	.8004	.9033	.7490	.8745	1.3351	.1255	10	0.9279
0.6458	37° 00′	.6018	9.7795	.7986	9.9023	.7536	9.8771	1.3270	0.1229	53° 00′	0.9250
0.6487	10	.6041	.7811	.7969	.9014	.7581	.8797	1.3190	.1203	50	0.9221
0.6516	20	.6065	.7828	.7951	.9004	.7627	.8824	1.3111	.1176	40	0.9192
0.6545	30	.6088	.7844	.7934	.8995	.7673	.8850	1.3032	.1150	30	0.9163
0.6574	40	.6111	.7861	.7916	.8985	.7720	.8876	1.2954	.1124	20	0.9134
0.6603	50	.6134	.7877	.7898	.8975	.7766	.8902	1.2876	.1098	10	0.9105
0.6632	38° 00′	.6157	9.7893	.7880	9.8965	.7813	9.8928	1.2799	0.1072	52° 00′	0.9076
0.6661	10	.6180	.7910	.7862	.8955	.7860	.8954	1.2723	.1046	50	0.9047
0.6690	20	.6202	.7926	.7844	.8945	.7907	.8980	1.2647	.1020	40	0.9018
0.6720	30	.6225	.7941	.7826	.8935	.7954	.9006	1.2572	.0994	30	0.8988
0.6749	40	.6248	.7957	.7808	.8925	.8002	.9032	1.2497	.0968	20	0.8959
0.6778	50	.6271	.7973	.7790	.8915	.8050	.9058	1.2423	.0942	10	0.8930
0.6807	39° 00′	.6293	9.7989	.7771	9.8905	.8098	9.9084	1.2349	0.0916	51° 00′	0.8901
0.6836	10	.6316	.8004	.7753	.8895	.8146	.9110	1.2276	.0890	50	0.8872
0.6865	20	.6338	.8020	.7735	.8884	.8195	.9135	1.2203	.0865	40	0.8843
0.6894	30	.6361	.8035	.7716	.8874	.8243	.9161	1.2131	.0839	30	0.8814
0.6923	40	.6383	.8050	.7698	.8864	.8292	.9187	1.2059	.0813	20	0.8785
0.6952	50	.6406	.8066	.7679	.8853	.8342	.9212	1.1988	.0788	10	0.8756
0.6981	40° 00′	.6428	9.8081	.7660	9.8843	.8391	9.9238	1.1918	0.0762	50° 00′	0.8727
0.7010	10	.6450	.8096	.7642	.8832	.8441	.9264	1.1847	.0736	50	0.8698
0.7039	20	.6472	.8111	.7623	.8821	.8491	.9289	1.1778	.0711	40	0.8668
0.7069	30	.6494	.8125	.7604	.8810	.8541	.9315	1.1708	.0685	30	0.8639
0.7098	40	.6517	.8140	.7585	.8800	.8591	.9341	1.1640	.0659	20	0.8610
0.7127	50	.6539	.8155	.7566	.8789	.8642	.9366	1.1571	.0634	10	0.8581
0.7156	41° 00′	.6561	9.8169	.7547	9.8778	.8693	9.9392	1.1504	0.0608	49° 00′	0.8552
0.7185	10	.6583	.8184	.7528	.8767	.8744	.9417	1.1436	.0583	50	0.8523
0.7214	20	.6604	.8198	.7509	.8756	.8796	.9443	1.1369	.0557	40	0.8494
0.7243	30	.6626	.8213	.7490	.8745	.8847	.9468	1.1303	.0532	30	0.8465
0.7272	40	.6648	.8227	.7470	.8733	.8899	.9494	1.1237	.0506	20	0.8436
0.7301	50	.6670	.8241	.7451	.8722	.8952	.9519	1.1171	.0481	10	0.8407
0.7330	42° 00′	.6691	9.8255	.7431	9.8711	.9004	9.9544	1.1106	0.0456	48° 00′	0.8378
0.7359	10	.6713	.8269	.7412	.8699	.9057	.9570	1.1041	.0430	50	0.8348
0.7389	20	.6734	.8283	.7392	.8688	.9110	.9595	1.0977	.0405	40	0.8319
0.7418	30	.6756	.8297	.7373	.8676	.9163	.9621	1.0913	.0379	30	0.8290
0.7447	40	.6777	.8311	.7353	.8665	.9217	.9646	1.0850	.0354	20	0.8261
0.7476	50	.6799	.8324	.7333	.8653	.9271	.9671	1.0786	.0329	10	0.8232
0.7505	43° 00′	.6820	9.8338	.7314	9.8641	.9325	9.9697	1.0724	0.0303	47° 00′	0.8203
0.7534	10	.6841	.8351	.7294	.8629	.9380	.9722	1.0661	.0278	50	0.8174
0.7563	20	.6862	.8365	.7274	.8618	.9435	.9747	1.0599	.0253	40	0.8145
0.7592	30	.6884	.8378	.7254	.8606	.9490	.9772	1.0538	.0228	30	0.8116
0.7621	40	.6905	.8391	.7234	.8594	.9545	.9798	1.0477	.0202	20	0.8087
0.7650	50	.6926	.8405	.7214	.8582	.9601	.9823	1.0416	.0177	10	0.8058
0.7679	44° 00′	.6947	9.8418	.7193	9.8569	.9657	9.9848	1.0355	0.0152	46° 00′	0.8029
0.7709	10	.6967	.8431	.7173	.8557	.9713	.9874	1.0295	.0126	50	0.7999
0.7738	20	.6988	.8444	.7153	.8545	.9770	.9899	1.0235	.0101	40	0.7970
0.7767	30	.7009	.8457	.7133	.8532	.9827	.9924	1.0176	.0076	30	0.7941
0.7796	40	.7030	.8469	.7112	.8520	.9884	.9949	1.0117	.0051	20	0.7912
0.7825	50	.7050	.8482	.7092	.8507	.9942	.9975	1.0058	.0025	10	0.7883
0.7854	45° 00′	.7071	9.8495	.7071	9.8495	1.0000	0.0000	1.0000	0.0000	45° 00′	0.7854
		Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.		
		COSINES.		SINES.		COTANGENTS.		TANGENTS.		DEGREES.	RADIANS.

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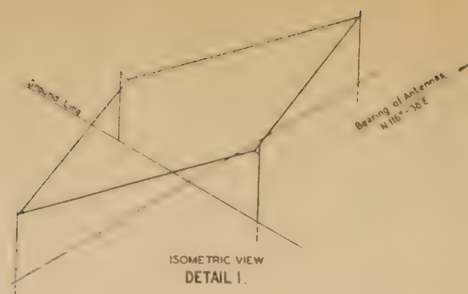
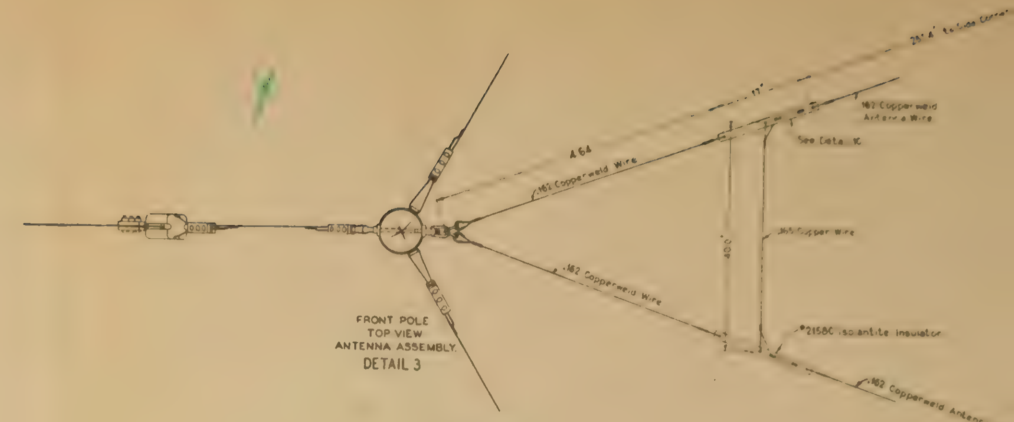
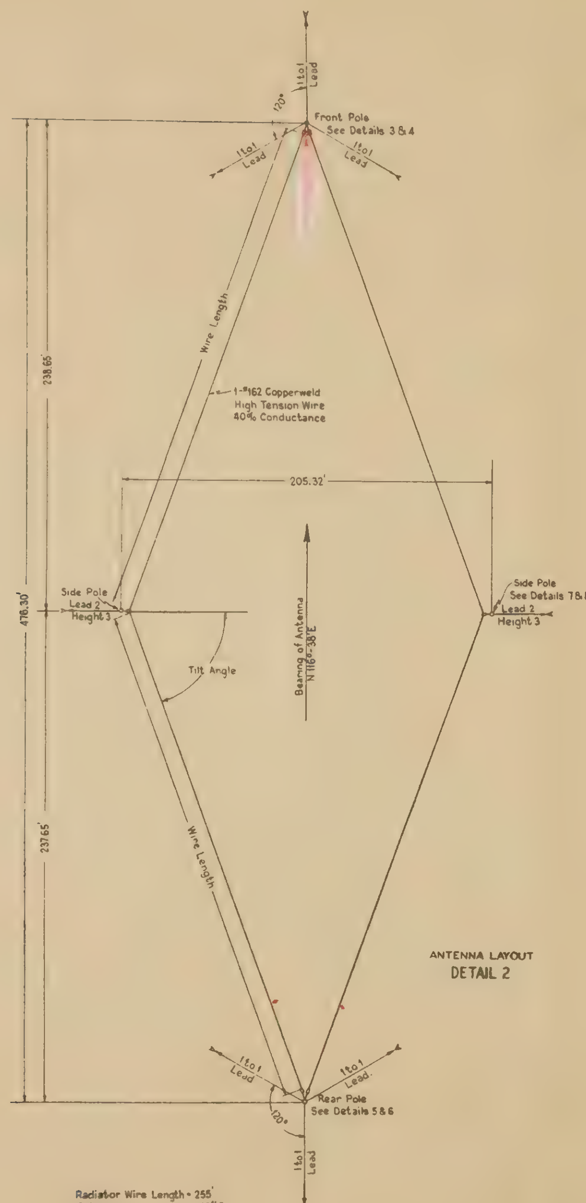


ISOMETRIC VIEW
DETAIL 1.

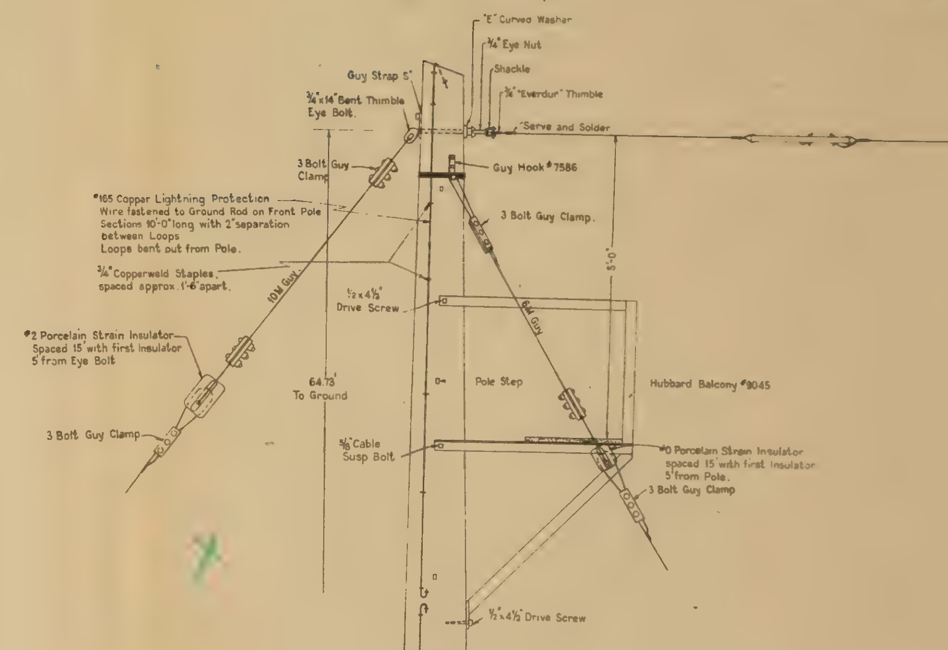


FRONT POLE
TOP VIEW
ANTENNA ASSEMBLY
DETAIL 3

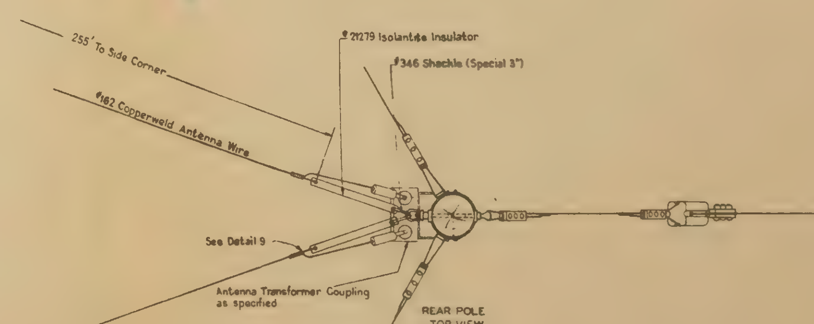


ANTENNA LAYOUT
DETAIL 2

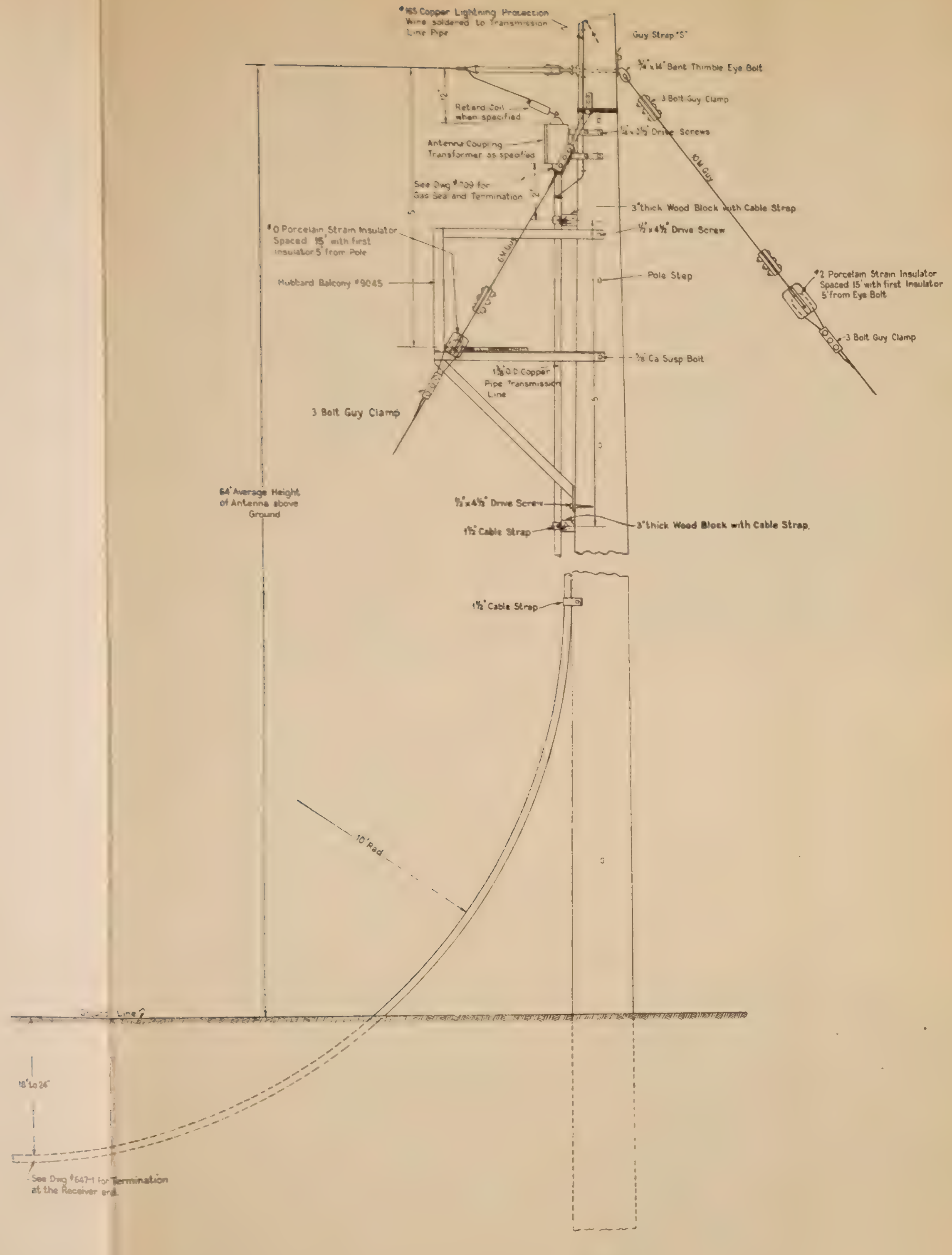
Radiator Wire Length = 255'
Tilt Angle = 67½°
Average Height of Antenna above Ground = 64'
Wire Sag = 1.56' at 60F.



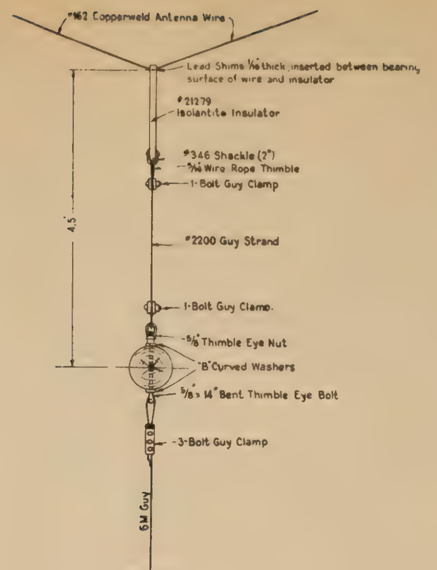
FRONT POLE
SIDE VIEW
ANTENNA ASSEMBLY
DETAIL 4.



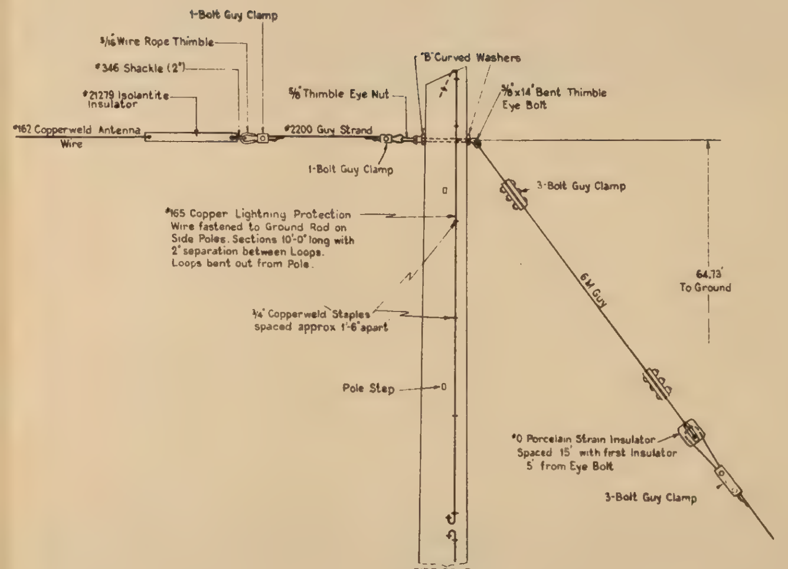
REAR POLE
TOP VIEW
ASSEMBLY.
DETAIL 5.



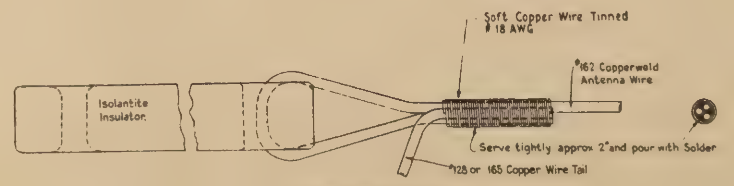
REAR POLE
SIDE VIEW
DETAIL 6



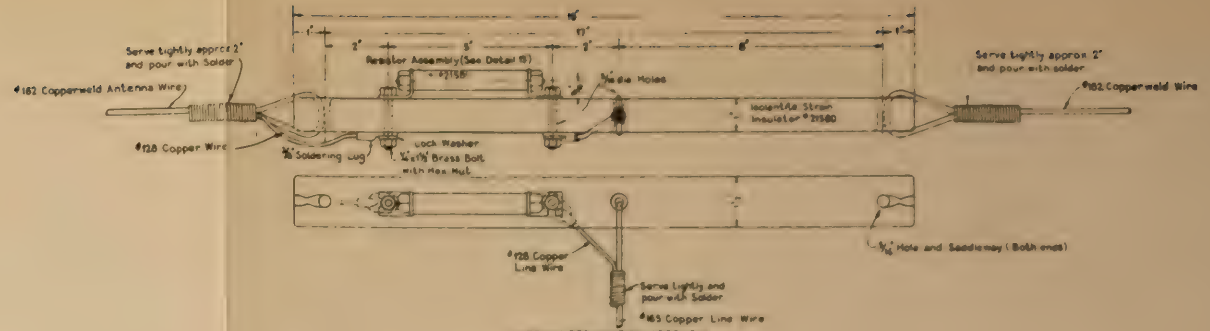
SIDE POLE
TOP VIEW
ASSEMBLY
DETAIL 7.



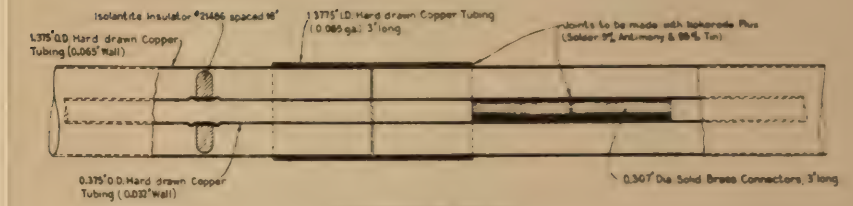
SIDE POLE
SIDE VIEW
ASSEMBLY
DETAIL 8



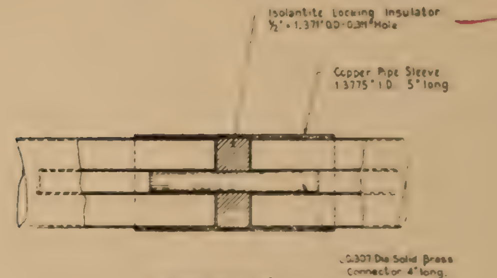
ANTENNA WIRE DEAD ENDS
DETAIL 9



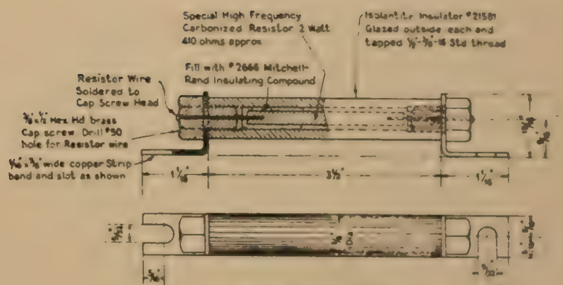
ANTENNA TERMINATION ASSEMBLY
DETAIL 10



TRANSMISSION LINE ASSEMBLY
DETAIL 11

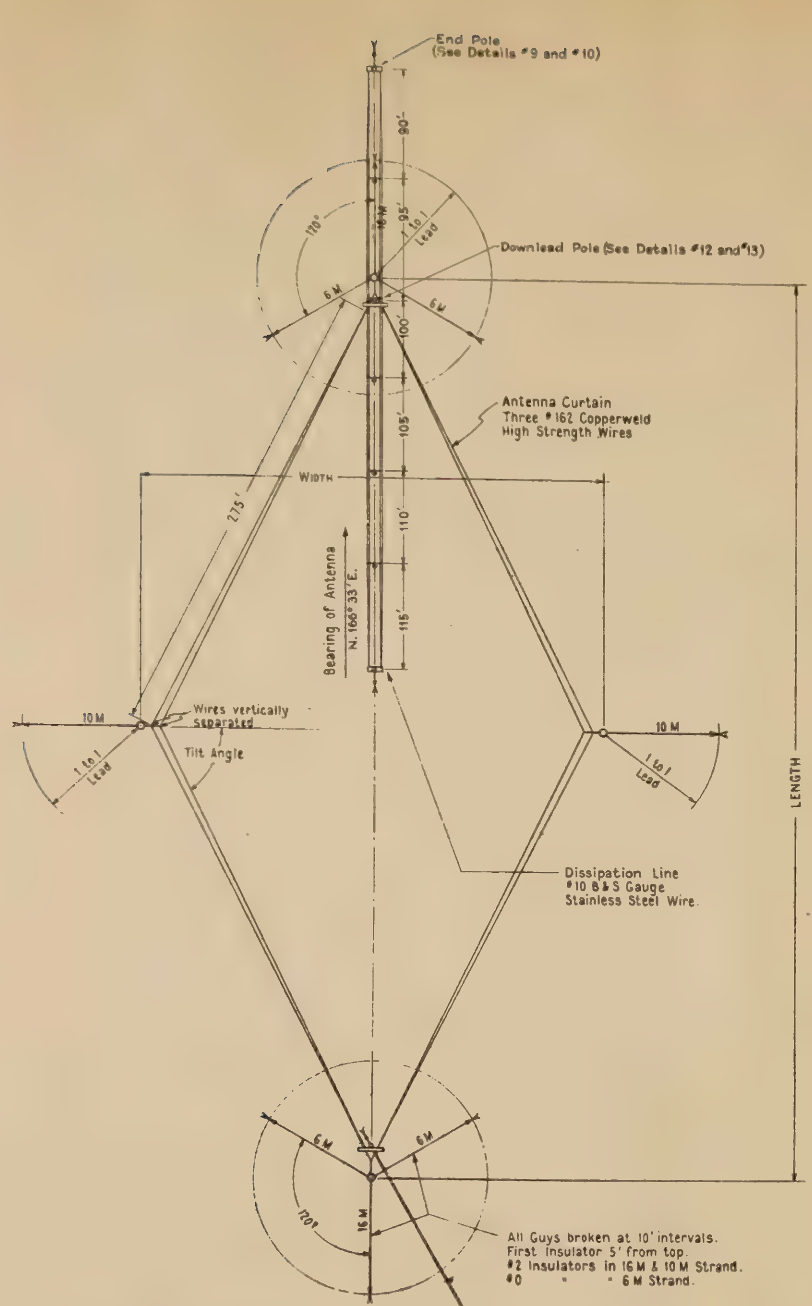


LOCKING INSULATOR
DETAIL 12



ANTENNA RESISTOR ASSEMBLY
DETAIL 13

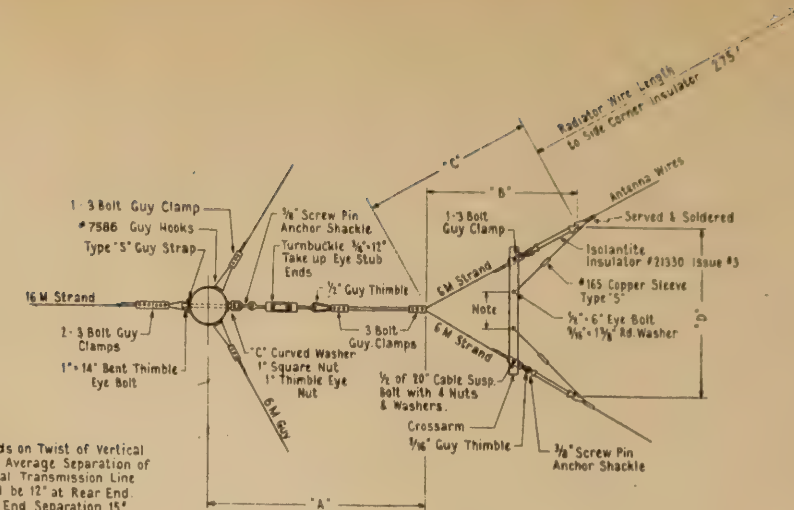
RECEIVING ANTENNA SYSTEMS
HORIZONTAL RHOMBIC ANTENNA DETAILS
FIG. 42



ANTENNA		
Pole to Pole Length	538.7	Ft.
Pole to Pole Width	201.0	Ft.
Radiator Lengths	275	Ft.
Vertical Separation of Wires at Side Poles	6.0	Ft.
Wire Sag at 60° F.	2.5	Ft.
Tilt Angle	70	°
Horizontal Separation of Wires at End Poles	3.0	Ft.

ANTENNA AND TERMINATION LINE LAYOUT

DET. 1

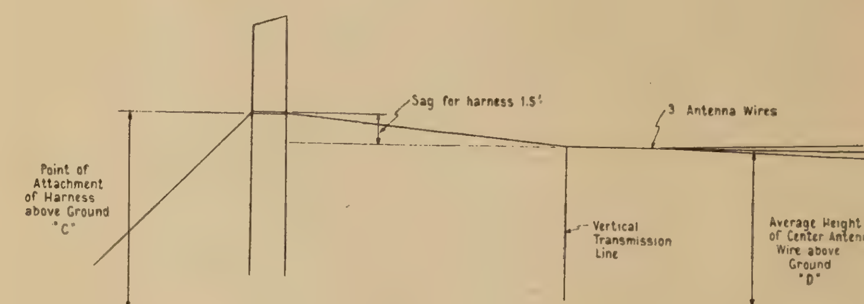


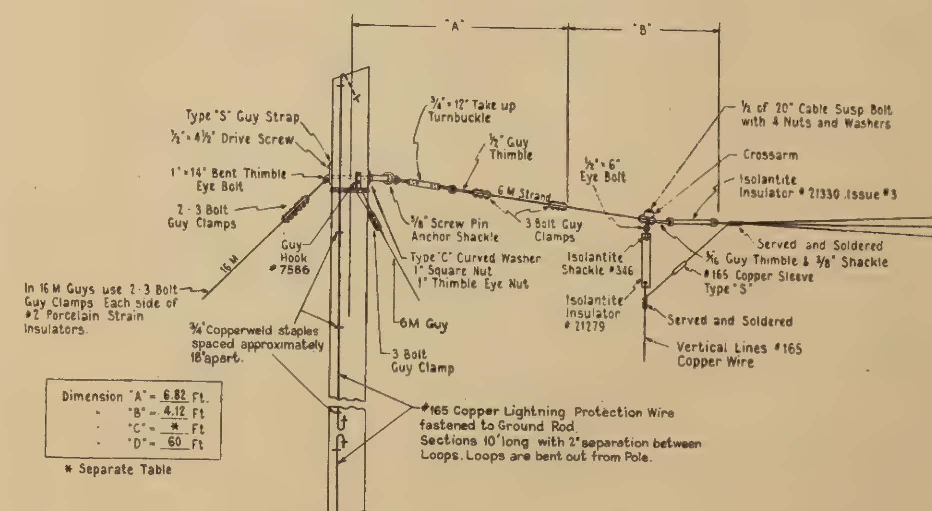
NOTE:
Depends on Twist of Vertical Wires Average Separation of Vertical Transmission Line should be 12" at Rear End. Front End Separation 15".

Dimension		
"A"	6.82	Ft
"B"	4.12	Ft
"C"	4.39	Ft
"D"	3.0	Ft

FRONT AND REAR END
ANTENNA ASSEMBLY
TOP VIEW

DET. 2



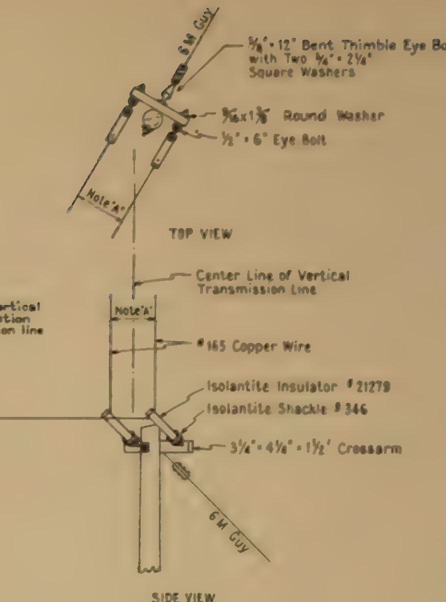


Dimension		
"A"	6.82	Ft.
"B"	4.12	Ft
"C"	*	Ft
"D"	60	Ft

* Separate Table

FRONT AND REAR END
ANTENNA ASSEMBLY
SIDE VIEW.

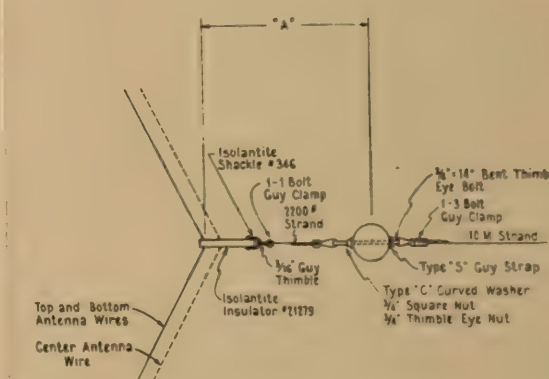
DET. 3



Note "A"
Depends on twist in vertical wires. Average separation of vertical transmission line should be 12".

REAR END
TRANSMISSION LINE ASSEMBLY
Scale: 1 in. = 2 Ft.

DET. 4



Dimension "A" = 4.35 Ft.

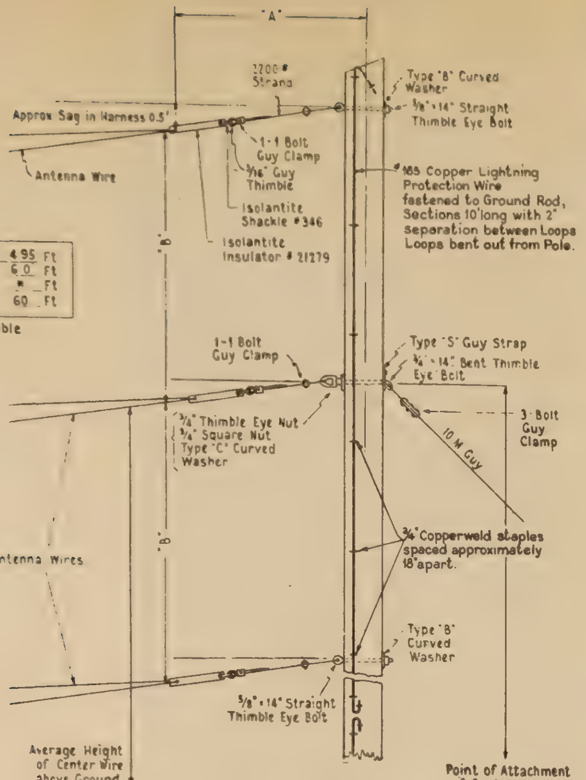
SIDE CORNER ASSEMBLY
Top View

DET. 5

Approx. Level of Horizontal Wires on Crossarms at adjacent poles.

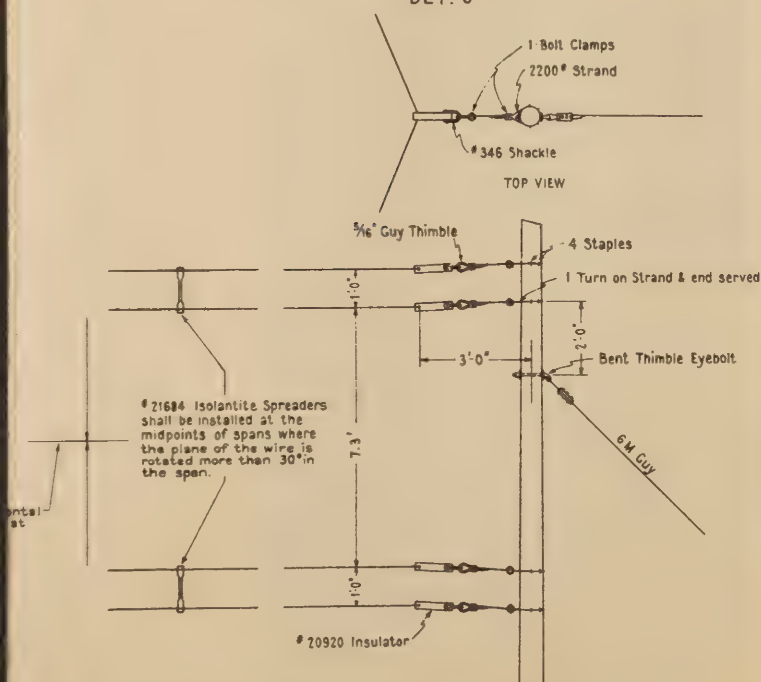
Dimension "A" =	4.95	Ft
" "B" =	6.0	Ft
" "C" =	*	Ft
" "D" =	60	Ft

* Separate Table



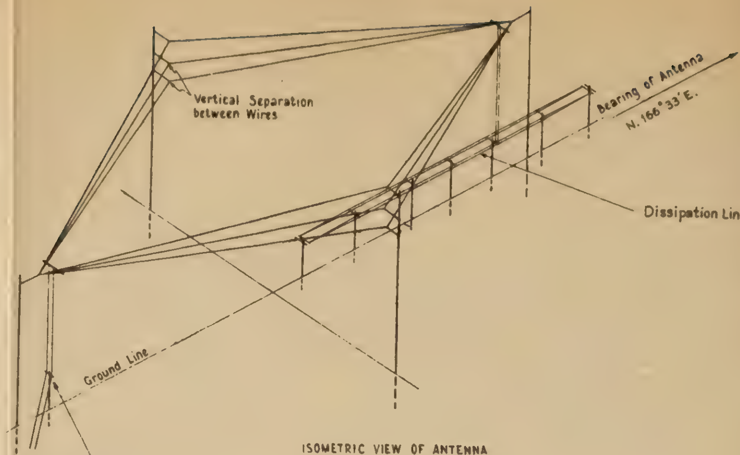
SIDE CORNER ASSEMBLY
SIDE VIEW

DET. 6



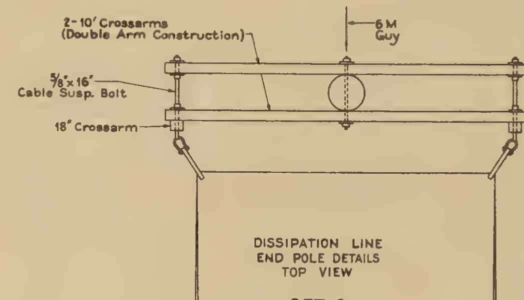
HARNESS DETAILS ON VERTICAL CORNER TRANSMISSION LINE
Scale: 1 in. = 2 ft.

DET. 7



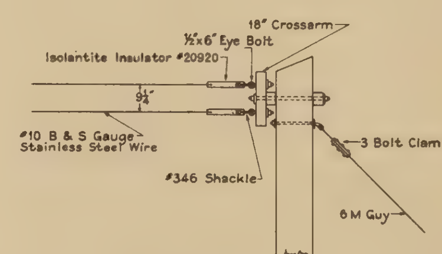
ISOMETRIC VIEW OF ANTENNA
Not to Scale

DET. 8



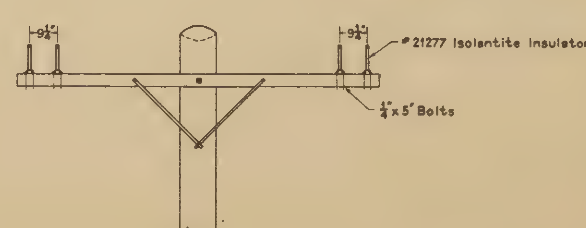
DISSIPATION LINE
END POLE DETAILS
TOP VIEW

DET. 9



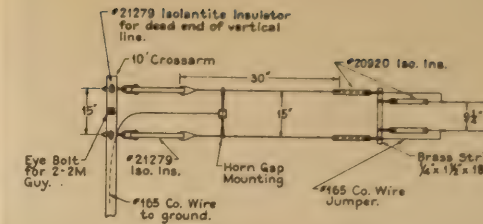
DISSIPATION LINE
END POLE DETAILS
SIDE VIEW

DET. 10



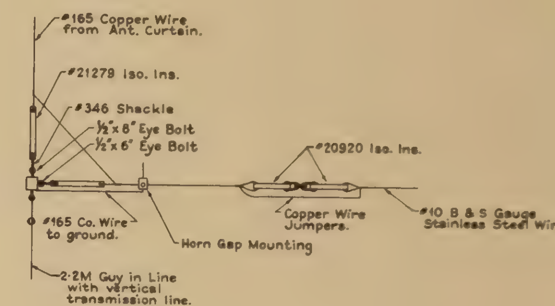
DISSIPATION LINE
INTERMEDIATE POLE-DETAILS

DET. 11



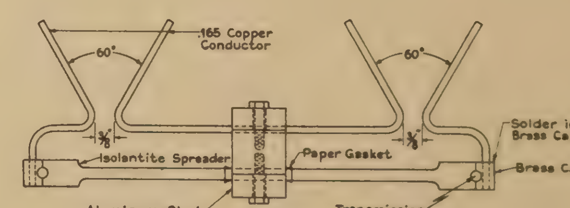
DISSIPATION LINE
DOWNLEAD POLE - DETAILS
TOP VIEW

DET. 12



DISSIPATION LINE
DOWNLEAD POLE DETAILS
SIDE VIEW

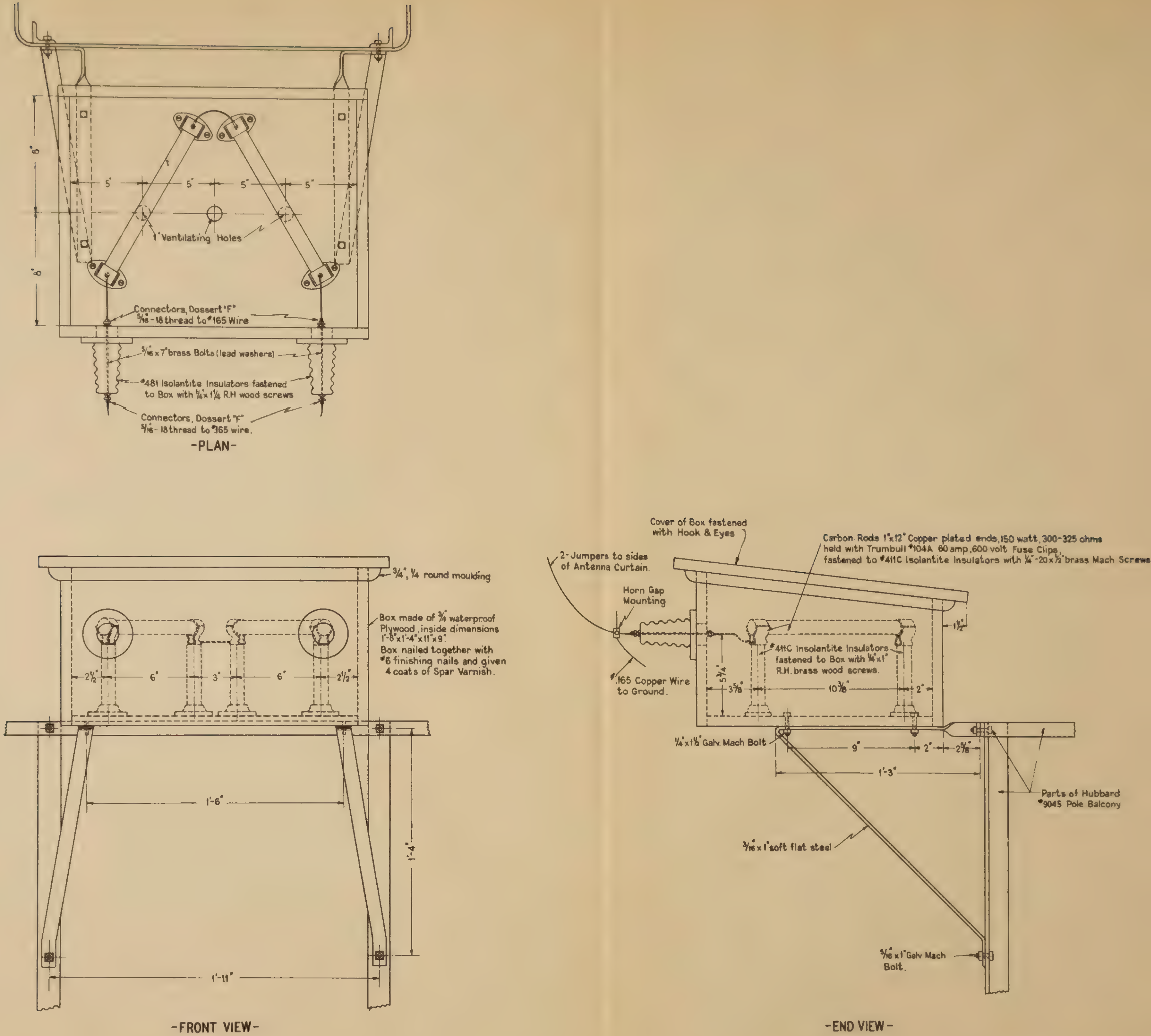
DET. 13



HORN GAP ASSEMBLY

DET. 14

HORIZONTAL RHOMBIC
TRANSMITTING ANTENNAS
FIG. 43



ANTENNA TERMINATION

MOUNTING ARRANGEMENT

FIG. 44